# 1.6A Inverses

Tuesday, August 4, 2026 7:55 AM

→ $A \cdot B = B \cdot A = I_{n \times n}$ then $B$ is inverse of $A$ [$A$ is $n \times n$] square

→ Inverse is unique

$\boxed{AB = I \quad \& \quad CA = I}$

$C(AB) = (CA)B$

$C I = I B$

$C = B$

→ $Ax = b$ $\qquad x = A^{-1}b$

↓

$\begin{bmatrix} x & & \\ & x & \\ & & x \end{bmatrix}$ ✓ $n$ pivots $\Longrightarrow$ Invertible

→ $(AB)^{-1} = B^{-1} \cdot A^{-1}$ $\qquad (AB)(B^{-1}A^{-1}) = I$

$(B^{-1}A^{-1})(AB) = I$

$(ABC \ldots Z)^{-1} = Z^{-1} \ldots B^{-1}A^{-1}$

→ matrix multiplication complexity $\qquad A \cdot B_{n \times n}$

$n^2 \times n$
$= n^3$

$\begin{bmatrix} \cdot & \end{bmatrix}_{n \times n}$

# G-J Method

$\boxed{Ax = b}$

$\boxed{AA^{-1} = I}$ need to find $A^{-1}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$n \times n \quad \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & & \cdots & 0 \\ \vdots & & & & & \\ 0 & 0 & - & - & & 1 \end{bmatrix}_{n \times n}$

$A \underbrace{(A^{-1}_{:1})}_{x_i} = \underbrace{\begin{matrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{matrix}}_{b_i}$

$A^{-1}_{:2} = \begin{matrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{matrix}$

$Ax_i = C_i$

$LC_i = b_i$

$\rightarrow [A \;\; e_1 \, e_2 \cdots e_n] =$

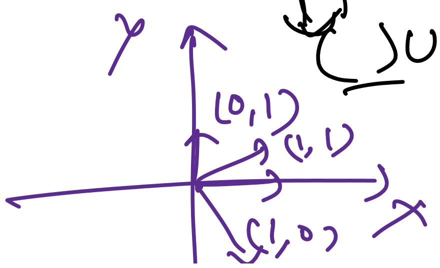
$e_i = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$\boxed{AA^{-1} = I}$

$[A \mid e_1 e_2 \ldots e_n]$

$(\quad) U$

$\rightarrow \begin{bmatrix} 2 & 1 & 1 & 1 & 0 & 0 \\ 4 & -6 & 0 & 0 & 1 & 0 \\ -2 & 7 & 2 & 0 & 0 & 1 \end{bmatrix}$



$$\left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & -8 & -2 & -2 & 1 & 0 \\ 0 & 8 & 3 & 1 & 0 & 1 \end{array}\right]$$

$(1,-1)$

$(E^{-1} F^{-1} G^{-1}) =$

$GFE = I$

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & -8 & -2 & -2 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$$

$U \quad x \qquad c \quad L^{-1}$

$L = E^{-1} F^{-1} G^{-1}$

$L^{-1} = G.F.E$

$(GFE)A \qquad (GFE) \cdot I = L^{-1}$

$\dfrac{U^{-1} L^{-1}}{A^{-1}}$

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & -8 & 0 & -4 & 3 & 2 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$$

$U$

$R1: R1 - R3$

$$\begin{array}{ccc|ccc} 2 & 1 & 0 & 2 & -1 & -1 \\ 0 & -8 & 0 & -4 & 3 & 2 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}$$

$R1 = R1 + \frac{1}{8} R2$

$$\left[\begin{array}{cccccc} 2 & 0 & 0 & 2-\frac{4}{8} & -1+\frac{3}{8} & -1+\frac{2}{8} \\ 0 & -8 & 0 & -4 & 3 & 2 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 2 & 0 & 0 & \frac{12}{8} & -\frac{5}{8} & -\frac{6}{8} \\ 0 & -8 & 0 & -4 & 3 & 2 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$$

$R1: R1/2$

$R2: R2/-8$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 12/16 & -8/16 & -6/16 \\ 0 & 1 & 0 & 4/8 & -3/8 & -7/8 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$$

$I$ $\qquad U^{-1}L^{-1}$

$U^{-1}U \rightarrow I$

$A = LU$

$U^{-1}L^{-1} = A^{-1}$

$$[A \mid I] \xrightarrow{L^{-1}} [U \mid L^{-1}] \xrightarrow{U^{-1}} [I \mid A^{-1}]$$