



Matrix Theory

01 System of Linear Equations

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The central question



భారతీయ సాంకేతిక విజ్ఞాన సంస్థ హైదరాబాద్
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What does a system of linear equations mean geometrically, algebraically, and computationally?

Let's work with an example



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$$x + 2y = 5$$

$$3x - y = 1$$

- What does it mean to solve the system?

Let's work with an example



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$$x + 2y = 5$$

$$3x - y = 1$$

- What does it mean to solve the system?
- Find the values of x and y that satisfy both the equations

The row picture



$$x + 2y = 5$$

$$3x - y = 1$$

- Consider each equation separately

The row picture



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$$x + 2y = 5$$

$$3x - y = 1$$

- Consider each equation separately
- What do they represent?

The row picture



$$x + 2y = 5$$

$$3x - y = 1$$

- Consider each equation separately
- What do they represent?
- What is the solution?

The row picture - let's solve geometrically



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The row picture



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- What are the possibilities for 2 variable case?

The row picture



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- What are the possibilities for 2 variable case?
 - Lines intersect:

The row picture



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- What are the possibilities for 2 variable case?
 - Lines intersect:
 - Distinct parallel lines:

The row picture



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- What are the possibilities for 2 variable case?
 - Lines intersect:
 - Distinct parallel lines:
 - Lines coincide:

The row picture



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$$x + 2y + z = 4$$

$$2x - y + z = 1$$

$$x + y - z = 2$$

- Let's consider 3 variables!

The row picture



$$x + 2y + z = 4$$

$$2x - y + z = 1$$

$$x + y - z = 2$$

- Let's consider 3 variables!
- What does each equation represent?

The row picture



$$x + 2y + z = 4$$

$$2x - y + z = 1$$

$$x + y - z = 2$$

- Let's consider 3 variables!
- What does each equation represent?
- How do they intersect?

The row picture



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- What are the possibilities for the 3 variable case?

$$u + v + w = 2$$

$$2u + \quad 3w = 5$$

$$3u + v + 4w = 6$$

The row picture



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- What are the possibilities for the 3 variable case?
- Case 1: Planes not parallel but don't intersect (inconsistent!)

$$u + v + w = 2$$

$$2u + \quad 3w = 5$$

$$3u + v + 4w = 6$$

The row picture



- Case 2: Planes intersect in a line (infinite solutions!)

$$u + v + w = 2$$

$$2u + \quad 3w = 5$$

$$3u + v + 4w = 7$$

The row picture



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- Case 3: Planes are parallel and distinct (inconsistent or no solutions!)

$$u + v + w = 2$$

$$u + v + w = 5$$

$$u + v + w = 7$$

The row picture



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- Case 4: Planes coincide (infinite solutions!)

$$u + v + w = 2$$

$$2u + 2v + 2w = 4$$

$$3u + 3v + 3w = 6$$

The row picture



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- Case 5: Two planes are parallel and the third is not (no solution!)

$$u + v + w = 2$$

$$2u + 2v + 2w = 5$$

$$3u + v + 4w = 7$$

The row picture



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The row picture is geometrically intuitive in two and three dimensions, but becomes difficult to visualize in higher dimensions.

The column picture



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$$x + 2y = 5$$

$$3x - y = 1$$

- The two separate equations are one vector equation!

The column picture



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- The problem now becomes to find the combination of LHS vectors that gives RHS one

The column picture



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- Each vector is a point (Descartes turned geometry into algebra!)

The column picture



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- Each vector is a point (Descartes turned geometry into algebra!)
- Basic operations on vectors: multiplying by numbers and adding vectors

The column picture



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- Each vector is a point (Descartes turned geometry into algebra!)
- Basic operations on vectors: multiplying by numbers and adding vectors
- The result of multiplying vectors by scalars and adding is called 'Linear combination'

The column picture



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The column picture attempts to find the right linear combination of LHS vectors to produce the one on the RHS.

The column picture



- When will this fail?

$$u + v + w = b_1$$

$$2u + \quad 3w = b_2$$

$$3u + v + 4w = b_3$$

The column picture



- When will this fail?

$$u + v + w = b_1$$

$$2u + \quad 3w = b_2$$

$$3u + v + 4w = b_3$$

- We can solve this for $\mathbf{b} = (2, 5, 7)$ but not for $\mathbf{b} = (2, 5, 6)$

The column picture

- When will this fail?

$$u + v + w = b_1$$

$$2u + \quad 3w = b_2$$

$$3u + v + 4w = b_3$$

- We can solve this for $\mathbf{b} = (2, 5, 7)$ but not for $\mathbf{b} = (2, 5, 6)$
- Why not?

The column picture



$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix}$$

- LHS columns lie on a plane

The column picture



$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix}$$

- LHS columns lie on a plane
- $\rightarrow$ Every combination of them lies on that plane (that passes through origin)

The column picture



$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix}$$

- LHS columns lie on a plane
- $\rightarrow$ Every combination of them lies on that plane (that passes through origin)
- If **b** does not lie on that plane $\rightarrow$

The column picture



$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix}$$

- LHS columns lie on a plane
- $\rightarrow$ Every combination of them lies on that plane (that passes through origin)
- If $\mathbf{b}$ does not lie on that plane $\rightarrow$
- If $\mathbf{b}$ lies on that plane $\rightarrow$

The column picture



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- How do we know if the vectors lie on a plane?

The column picture



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- How do we know if the vectors lie on a plane?
- One way: find a combination of them that gives zero

Summary



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- Row Picture: Intersection of planes
 - Column picture: Combination of columns
-
- If rows combine to give zero $\rightarrow$ columns would also do
 - If n planes have no point in common (or infinite points), then the columns lie on the same plane!



- Gaussian Elimination: An intuitive algorithm to solve the system of linear equations

Rough work



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