Foundations of Machine Learning AI2000 and AI5000

FoML-15
Linear Discriminant Functions

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July-Nov 2025

भारतीय सांकेतिक विज्ञान संस्था हैदराबाद्
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So far in FoML

- Intro to ML and Probability refresher

- MLE, MAP, and fully Bayesian treatment

- Linear Regression with basis functions (regularization, model selection

- Bias-Variance Decomposition (Bayesian Regression)

- Decision Theory - three broad classification strategies

- Probabilistic Generative Models - Continuous & discrete data

Discriminant Functions

Discriminant

- Function that takes an input and assigns one of the classes as output

- Restrict to '*Linear Discriminants*'
  - Decision surfaces are hyperplanes

Discriminant Functions - two classes

- Input $x \in \mathbb{R}^D$

- Targets $t \in \{C_1, C_2\}$

- Discriminant Function

$$y(\mathbf{x}) = f(\tilde{\mathbf{w}}^T \phi(\mathbf{x}))$$

$$\phi(\mathbf{x}) = (\phi_0(\mathbf{x}), \ldots, \phi_{M-1}(\mathbf{x}))^T$$

Linear Decision boundaries   Generalized Linear Models

భారతీయ సాంకేతిక విజ్ఞాన సంస్థ హైదరాబాద్
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Discriminant Functions - two classes

- Simplest discriminant function $\quad y(\mathbf{x}, \tilde{\mathbf{w}}) = \mathbf{w}^T \mathbf{x} + w_0$

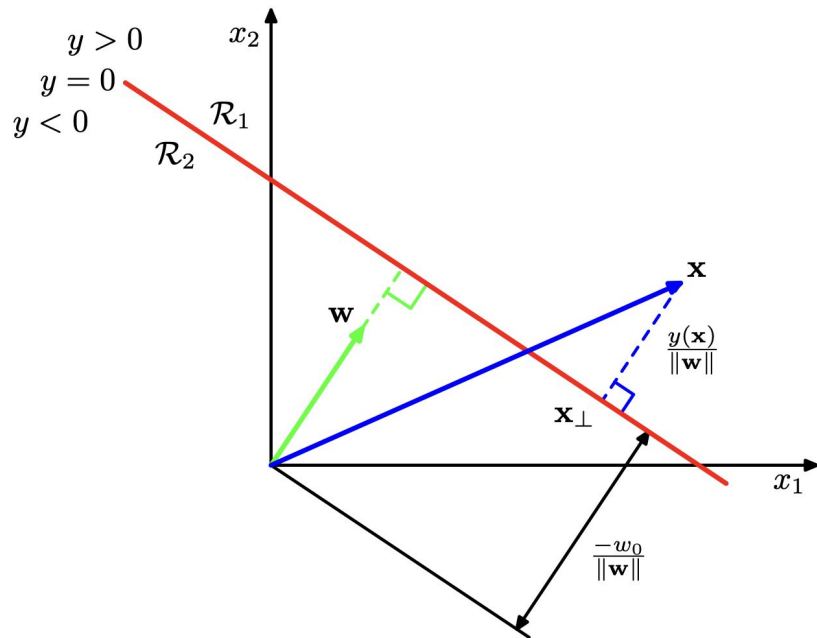- Decision boundary $\quad y(\mathbf{x}, \tilde{\mathbf{w}}) = 0$

Discriminant Functions - two classes

W determines the orientation of the decision boundary
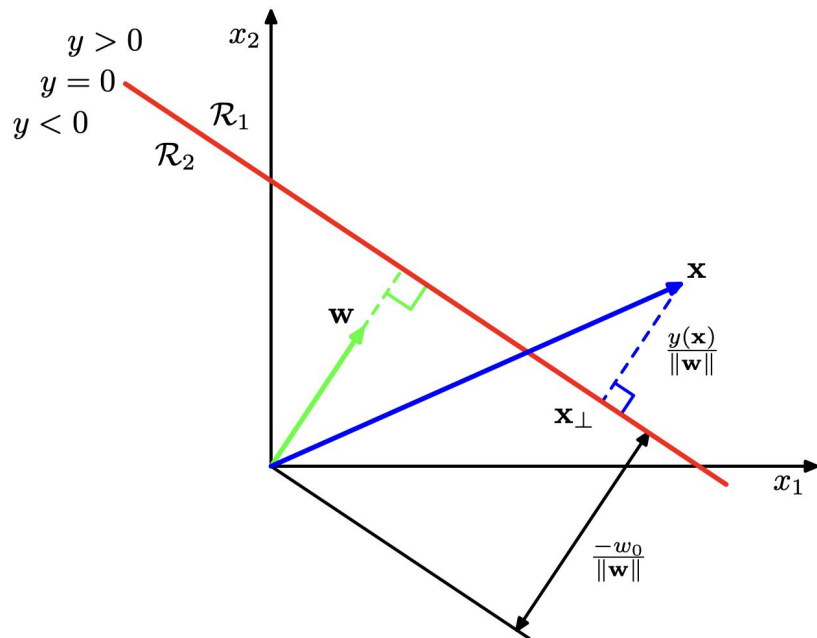
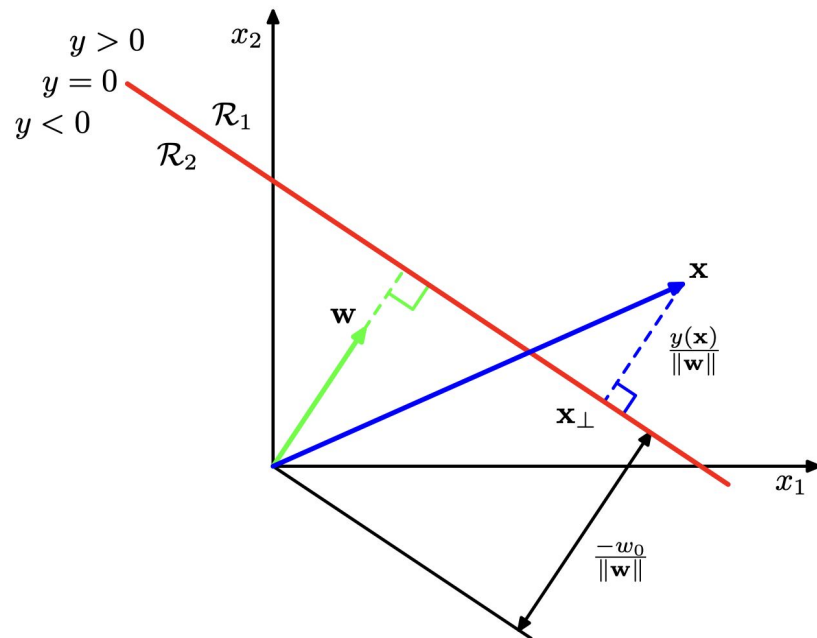# Discriminant Functions - two classes

Normal distance from origin

W_0 shifts the boundary away from origin
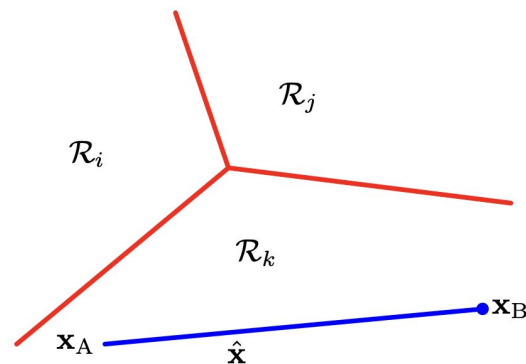
Discriminant Functions - two classes

y(x) gives the signed perpendicular distance from the boundary

Discriminant Functions: Multiple Classes

- K-class discriminant $y_k(\mathbf{x}) = \mathbf{w}_k{}^T\mathbf{x} + w_{k0}$

- Class assignment
  - to C_k if

- Decision boundary:

- Decision regions (for GLM) are convex

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Next
Least Squares for Classification