Foundations of Machine Learning
AI2000 and AI5000

FoML-03
Probability - Expectation, Variance and Gaussian Distribution

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Data-driven Intelligence & Learning Lab

So far in FoML

- What is ML and the learning paradigms
- Probability refresher
  - Sum rule, product rule, Random variables, Bayes Theorem, Independence

Expectation, Variance and the Gaussian Distribution

Expectation

- Random variable X and a function $f: X \rightarrow \mathbb{R}$

$$\mathbb{E}[f] = \mathbb{E}_{x \sim p(X)}[f(x)]$$

Expectation

- For N points drawn from p(X)

$$\mathbb{E}[f] =$$

Expectation

- Conditional expectation

$$\mathbb{E}[f/y] = \mathbb{E}_{x \sim p(X/Y=y)}[f(x)]$$

Variance

- Expected quadratic distance between f and its mean $\mathbb{E}[f]$

$$\mathrm{var}(f)$$

Covariance

- Measures the extent to which two random variables X and Y vary together

$$\mathrm{cov}[X, Y]$$

Covariance

- X and Y are vectors of random variables
- Covariance matrix

$$\text{cov}[X, Y]$$

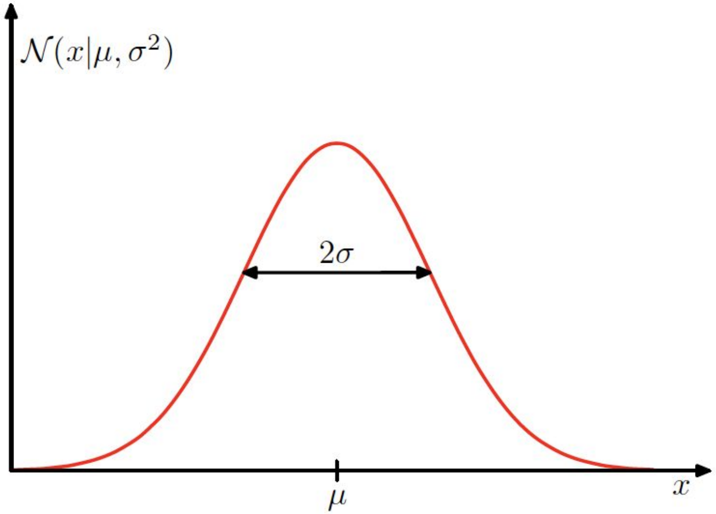
Covariance

- Between independent variables

$$\mathrm{cov}[X, Y]$$

Gaussian Distribution

Gaussian Distribution



$$\mathcal{N}(x|\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$$

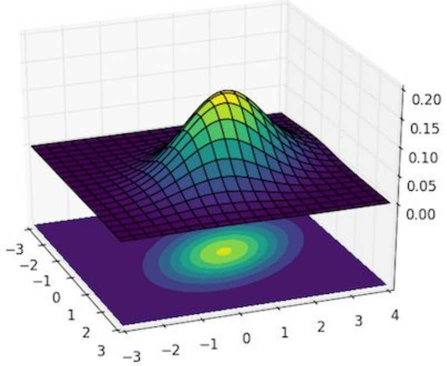
$$x \sim \mathcal{N}(x|\mu,\sigma^2)$$

$$\mathbb{E}[x] = \mu \qquad \mathrm{Var}(x) = \sigma^2$$

Multivariate Gaussian Distribution

- D-dimensional vector $\mathbf{x} = (x_1, x_2, \ldots x_D)^T$

$$\mathcal{N}(\mathbf{x}|\mu, \Sigma) = \frac{1}{(2\pi)^{D/2}|\mathbf{\Sigma}|^{1/2}} \exp\left(-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^\top \mathbf{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right)$$



Next
Maximum Likelihood Principle