Deep Learning for Computer Vision

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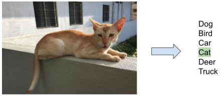
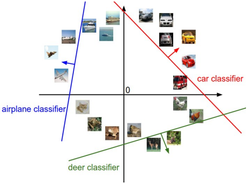
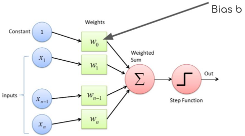
So far in the class..

- Image classification and Linear Classifier
- Perceptron







Recap: Linear classifier

1. $f(x) = \sigma(\mathbf{w}^T\mathbf{x} + b)$

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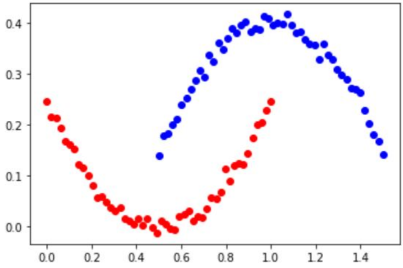
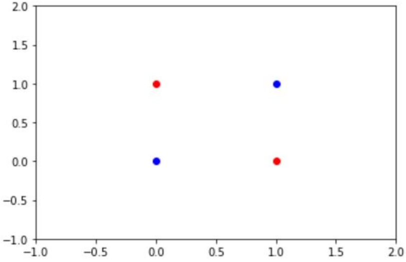
2. Seen a couple of simple examples: MP neuron and Perceptron

Linear Classifiers: Shortcomings

- Lower capacity: data has to be linearly separable
- Some times no hyper-plane can separate the data (e.g. XOR data)





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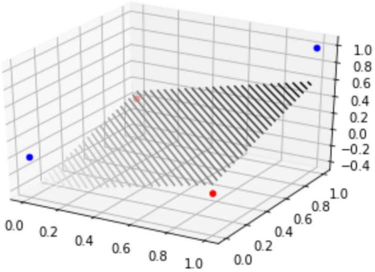
2. Consider the xor case
   $\phi(\mathbf{x}) = \phi(x_u, x_v) = (x_u, x_v, x_u x_v)$
3. Consider the perceptron in the new space $f(\mathbf{x}) = \sigma(\mathbf{w}^T \phi(\mathbf{x}) + b)$



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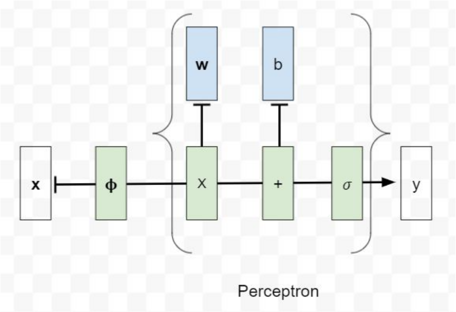

Pre-processing

1. Feature design (or pre-processing) may also be another way to reduce the capacity without affecting (or improving) the bias

Extending Linear Classifier

1. Single class: $f(\mathbf{x}) = \sigma(\mathbf{w}^T\mathbf{x} + b)$ from $\mathcal{R}^D \rightarrow \mathcal{R}$ where $\mathbf{w}$ and $\mathbf{x} \in \mathcal{R}^D$

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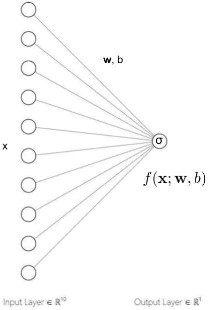
2. Multi-class: $f(\mathbf{x}) = \sigma(\mathbf{W}\mathbf{x} + \mathbf{b})$ from $\mathcal{R}^D \rightarrow \mathcal{R}^C$ where $\mathbf{W} \in \mathcal{R}^{C \times D}$ and $\mathbf{b} \in \mathcal{R}^C$

Single unit to a layer of Perceptrons



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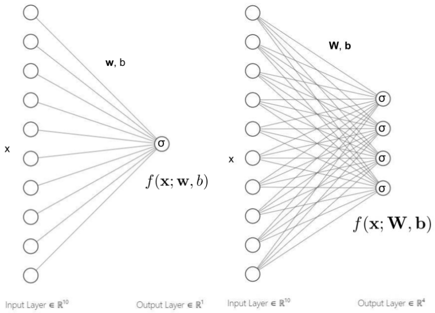

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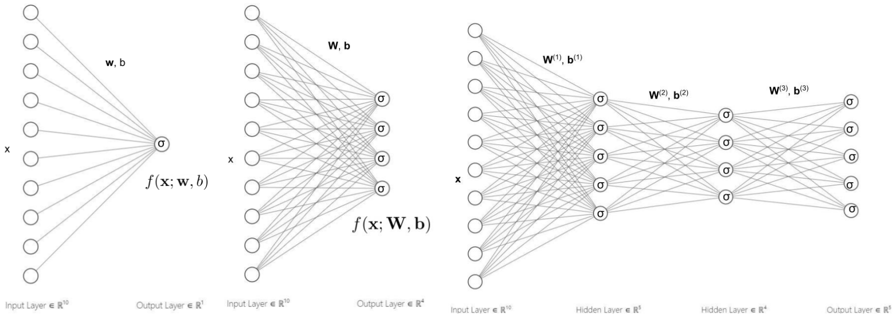

Formal Representation

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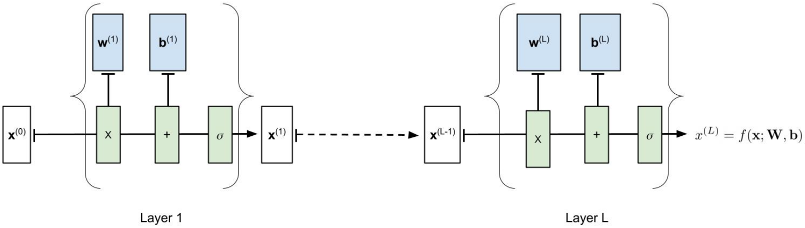
2. can be represented as:
   $\mathbf{x}^{(0)} = \mathbf{x}$,
   $\forall l = 1, \ldots, L, \quad \mathbf{x}^{(l)} = \sigma(\mathbf{W}^{(l)^T}\mathbf{x}^{(l-1)} + \mathbf{b}^{(l)})$, and

MLP



$\mathbf{w}^{(1)}$
$\mathbf{b}^{(1)}$
$\mathbf{x}^{(0)}$
X
+
σ
$\mathbf{x}^{(1)}$
$\mathbf{x}^{(L-1)}$
$\mathbf{w}^{(L)}$
$\mathbf{b}^{(L)}$
X
+
σ
$x^{(L)} = f(\mathbf{x}; \mathbf{W}, \mathbf{b})$
Layer 1
Layer L

Nonlinear Activation

1. Note that σ is nonlinear

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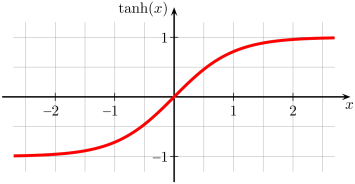
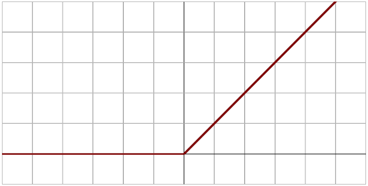
2. If it is an affine function, the full MLP becomes a complex affine transformation (composition of a series of affine mappings)

Nonlinear Activation

Familiar activation functions





Hyperbolic Tangent (Tanh) $x \to \frac{2}{1+e^{-2x}} - 1$ and
Rectified Linear Unit (ReLU) $x \to \max(0, x)$ respectively

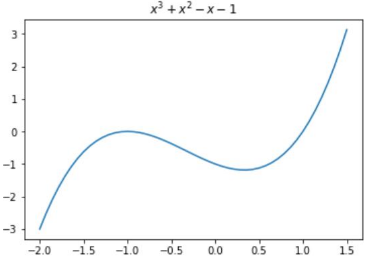
Universal Approximation using ReLU functions

1. We can approximate any function f from $[a, b]$ to $\mathcal{R}$ with a linear combination of ReLU functions

Example credits: Brendan Fortuner, and https://towardsdatascience.com/

Universal Approximation using ReLU functions

1. We can approximate any function f from $[a, b]$ to $\mathcal{R}$ with a linear combination of ReLU functions
2. Let's approximate the following function using a bunch of ReLUs:



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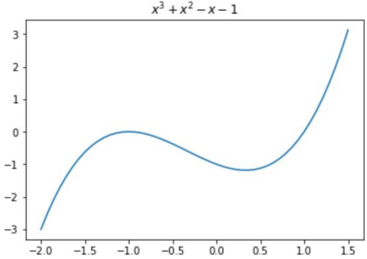
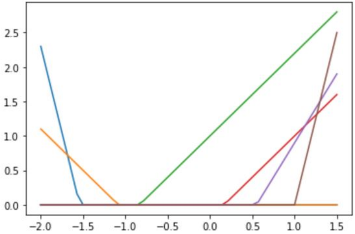
Universal Approximation using ReLU functions

1. $n_1 = ReLU(-5x - 7.7), n_2 = ReLU(-1.2x - 1.3), n_3 = ReLU(1.2x + 1), n_4 = ReLU(1.2x - 0.2), n_5 = ReLU(2x - 1.1), n_6 = ReLU(5x - 5)$





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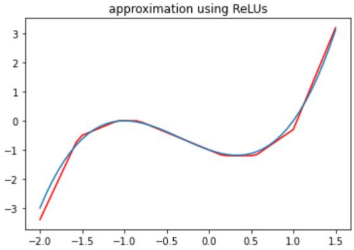
Universal Approximation using ReLU functions

1. Appropriate combination of these ReLUs:
   $-n_1 - n_2 - n_3 + n_4 + n_5 + n_6$



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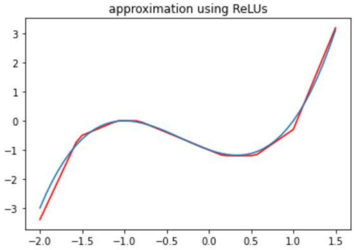
2. Note that this also holds in case of other activation functions with mild assumptions.



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2. $\mathbf{x} \to \mathbf{w}^T \sigma(W\mathbf{x} + \mathbf{b})$
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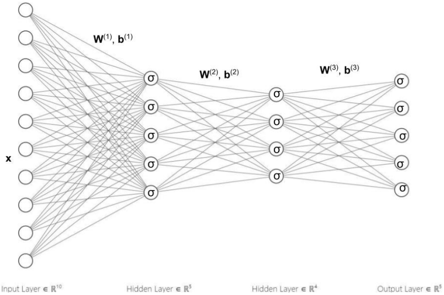
3. However, the resulting NN
   - May require infeasible size for the hidden layer
   - May not generalize well

MLP for regression

1. Output is a continuous variable in $\mathcal{R}^D$
   - Output layer has that many perceptrons (When $D = 1$, regresses a scalar value)
   - May employ a squared error loss



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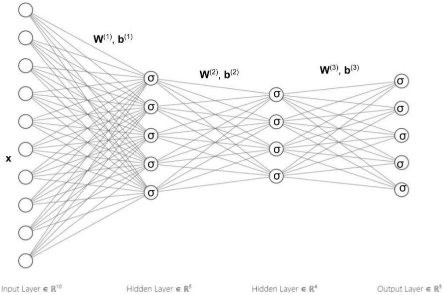
2. Can have an arbitrary depth (number of layers)



MLP for classification

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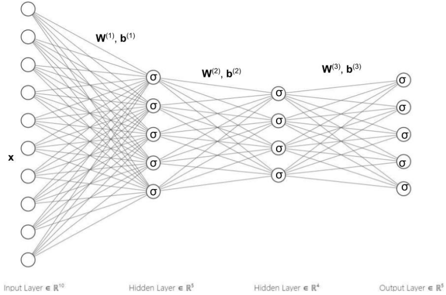

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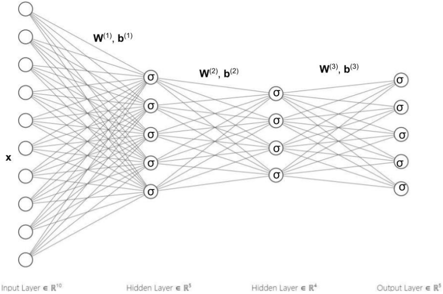

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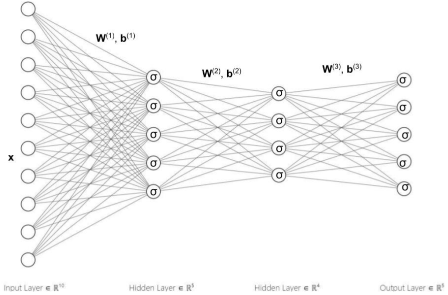

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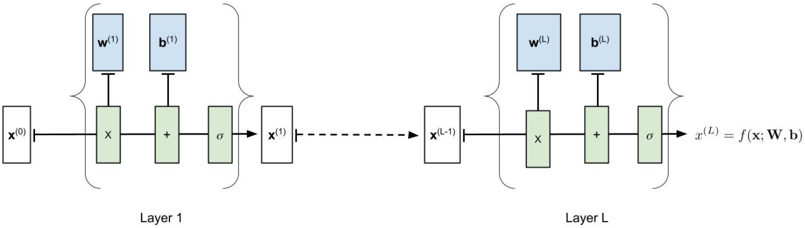
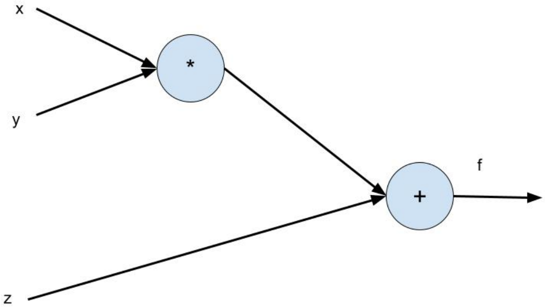
- specific to each loss function :-(

Solution: Computational graphs



E.g. Computational graph

$$f(x, y, z) = xy + z$$



E.g. Computational graph: Forward pass

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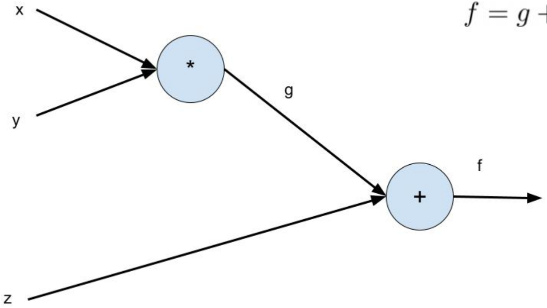

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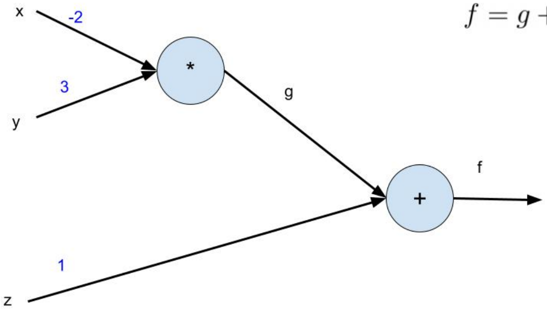

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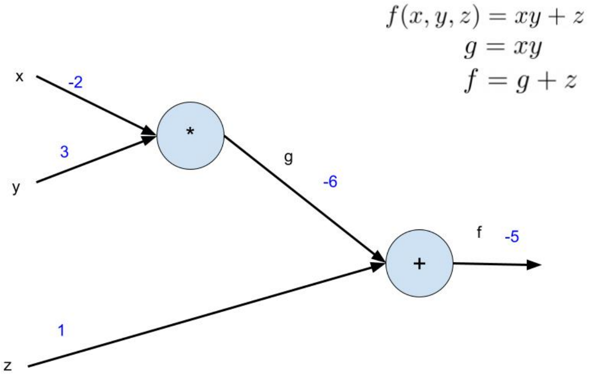

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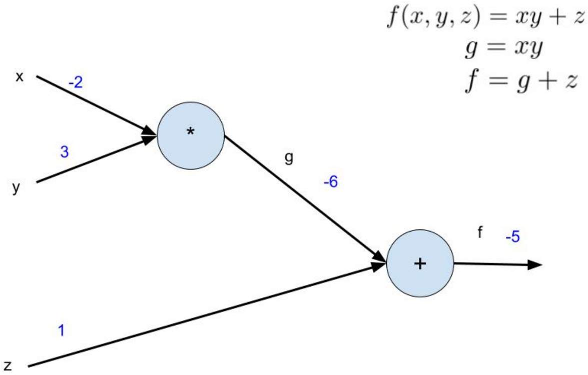

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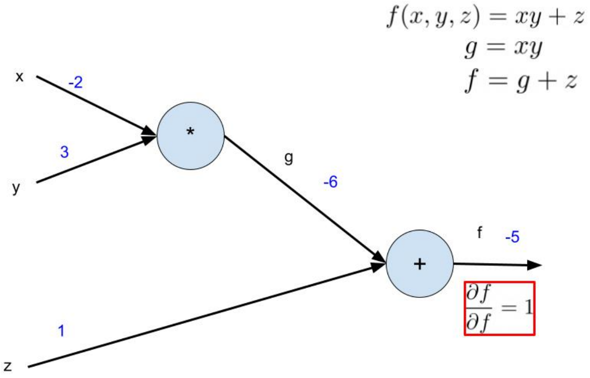

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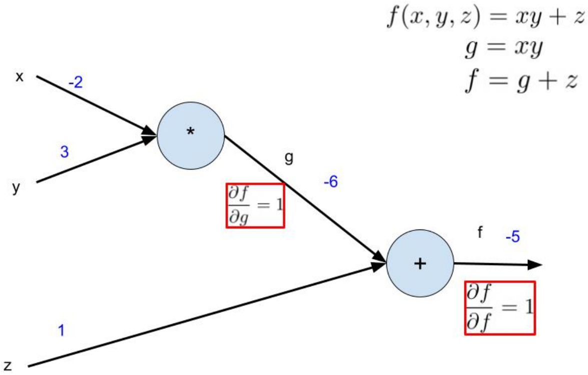

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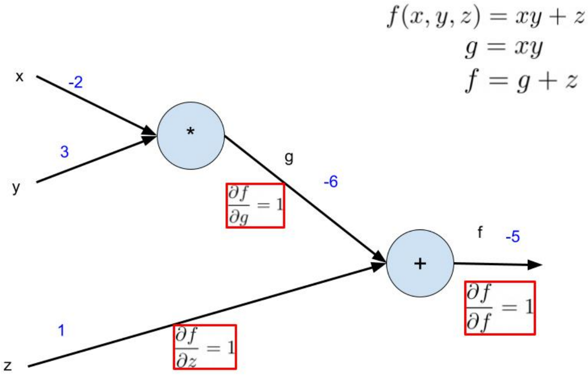

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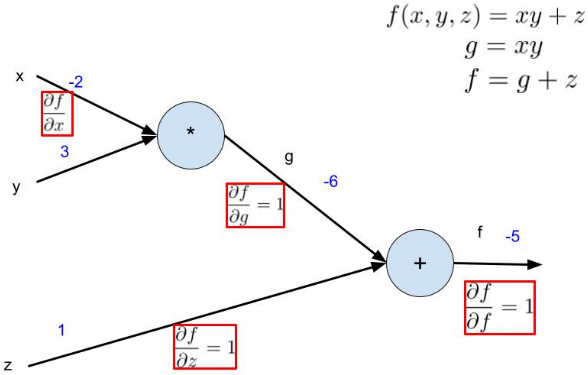

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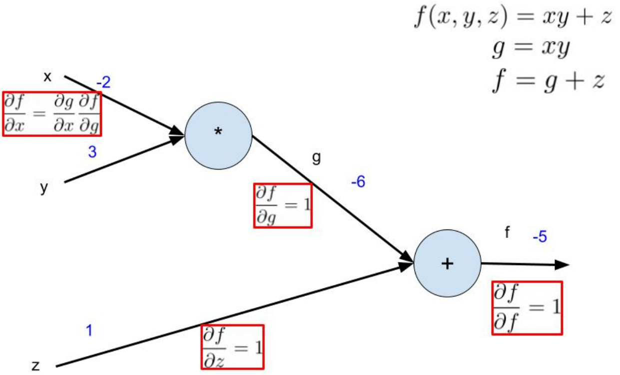

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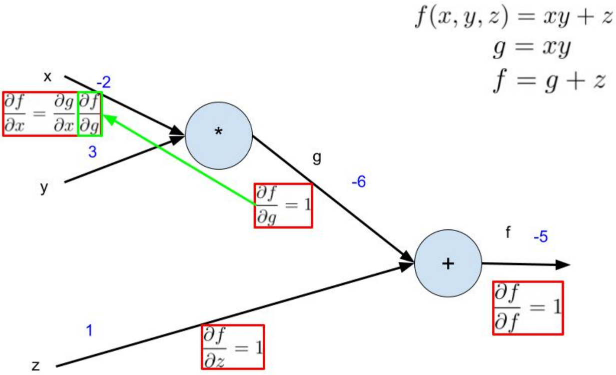

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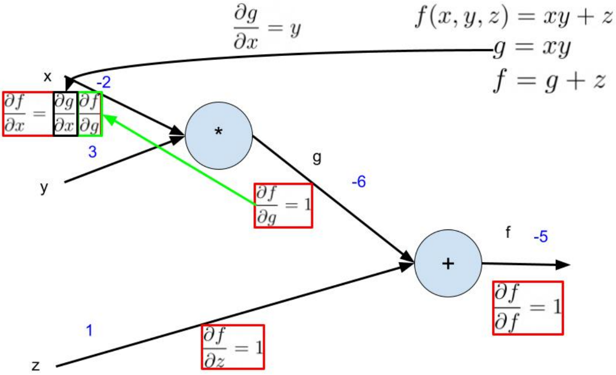

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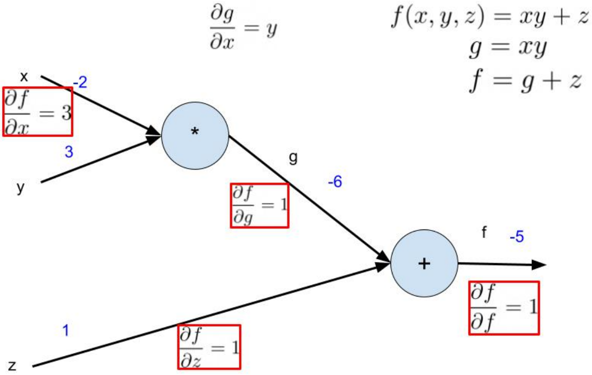

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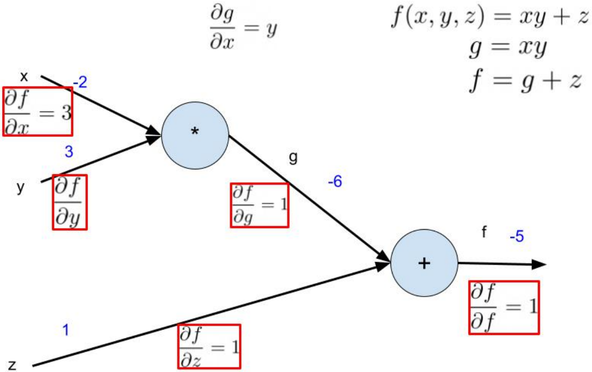

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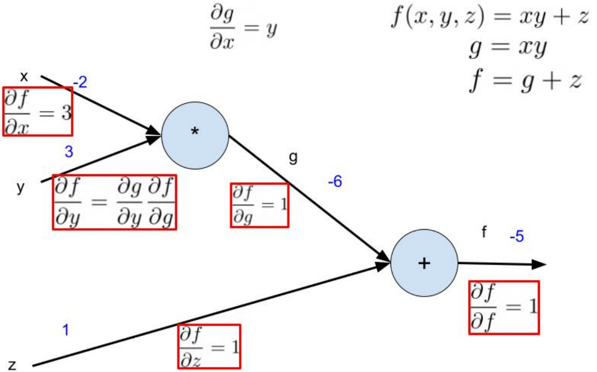

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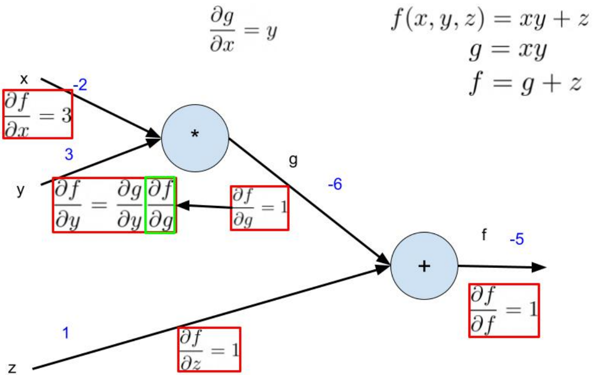

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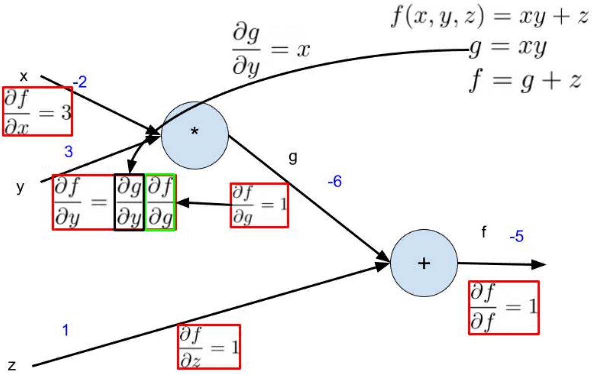

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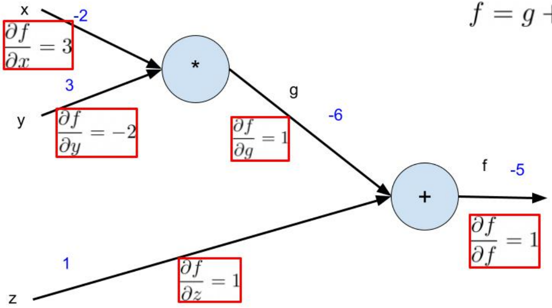
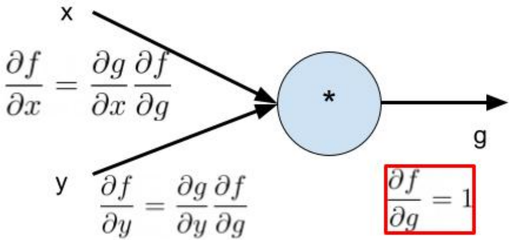


Gradient flow

- down steam gradient = local gradient × upstream gradient



$g_d = g_l \times g_u$

References

1. Cybenko G. 1989, Approximation by superpositions of a sigmoidal function