Deep Learning

17 Autoencoders

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Representation Learning

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2. to learn representations of data
3. One way to do so is through Autoencoers (or, auto-associative neural networks)

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2. These applications require to learn the meaningful degrees of freedom that constitute the signal
3. Typically, these degrees of freedom are of lesser dimensions than the signal

Example: Synthesizing Human faces

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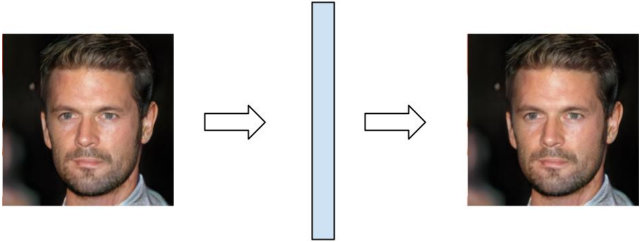
   - skull size and shape
   - color of skin and eyes
   - features of nose and lips, etc.
2. Even a comprehensive list of such things will be less than the number of pixels in the image (i.e. resolution)

Example: Synthesizing Human faces

1. If we can model these relatively small number of dimensions, we can synthesize a face with thousands of dimensions



Autoencoder: architecture

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3. Network consists of two parts: encoder (`f`) and decoder (`g`)

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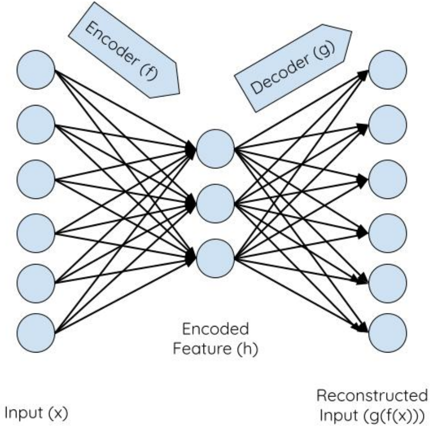

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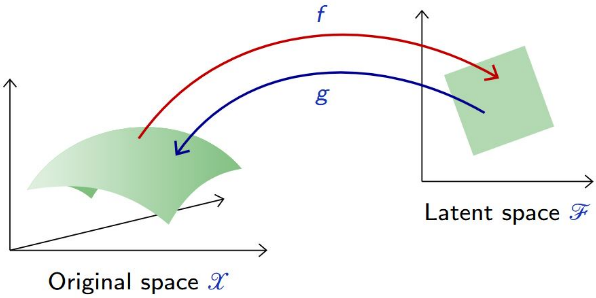

Figure credits: Francois Flueret

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2. Training: finding the parameters for the encoder $(f(\cdot; w_f))$ and decoder $(g(\cdot; w_g))$ optimizing the empirical loss

$$\hat{w}_f, \hat{w}_g = \underset{w_f, w_g}{\operatorname{argmin}} \frac{1}{N} \sum_n \|x_n - g(f(x_n; w_f); w_g)\|^2$$

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2. Hence, we may use BCE loss for training

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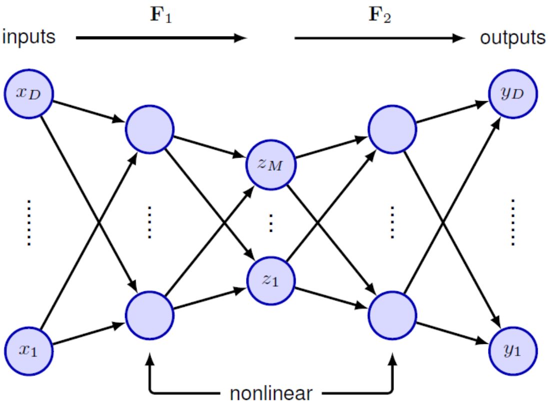
1. f and g are linear functions (data is normalized $x_i = \frac{1}{\sqrt{|X|}}(x_i - \mu)$) $\rightarrow$ optimal solution is PCA
2. Better results can be made possible with sophisticated transformations such as deep neural networks $\rightarrow$ Deep Autoencoders

Deep Autoencoders



Deep Autoencoders

Top row: original data samples
Bottom row: corresponding reconstructed samples (single ReLU layer of dimension 32)
Figure credits: Keras blog

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5. Tie the weights, i.e., $w_g = w_f^T$

Besides dimensionality reduction

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2. This can help to restore the missing components from an input

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4. This is referred to as a **Denoising Autoencoder**

Denoising Autoencoder

1. This can be illustrated with an additive Gaussian noise in case of a 2D signal and MSE

$$\hat{w} = \underset{w}{\operatorname{argmin}} \frac{1}{N} \sum_{n=1}^{N} \|x_n - \phi(x_n + \epsilon_n; w)\|^2 ,$$

where x_n are data samples and ϵ_n are Gaussian random noise

Denoising Autoencoder

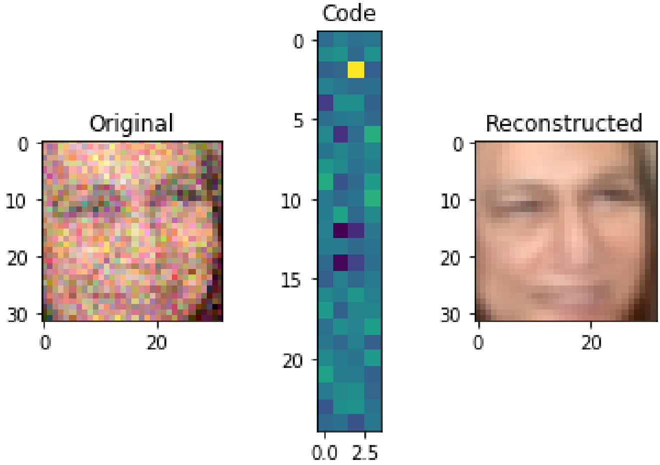

Figure credits: Ali Abdelal, https://stackabuse.com/

Masked Autoencoder

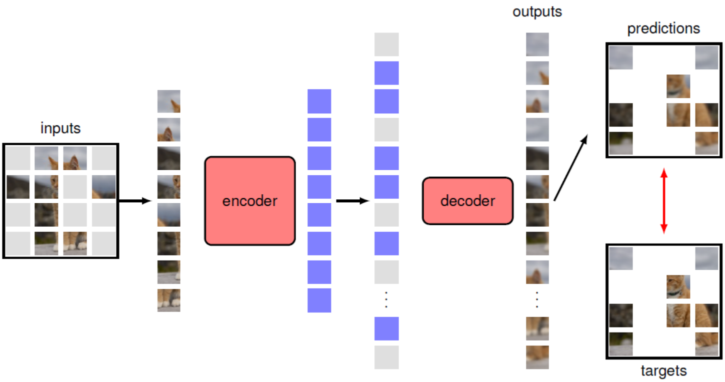

Figure credits: Bishop's book

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4. Enforces the mean neuron activation $(\hat{\rho}_l)$ to be close to ρ

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3. k - dimension of hidden layer
   m -size of training dataset

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3. Competition (in the latent/hidden layer) b/w 'being sensitive' and 'not sensitive' to the i/p variations
4. Ends up capturing only the important variations in the i/p (something like PCA)

Latent Representations

భారతీయ సాంకేతిక విజ్ఞాన సంస్థ హైదరాబాద్
भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

1. Consider two samples in the latent space and reconstruct the samples along the line joining these

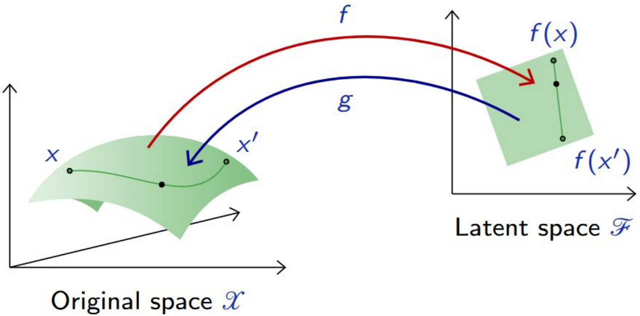

Figure credits: Francois Fleuret

Latent Representations

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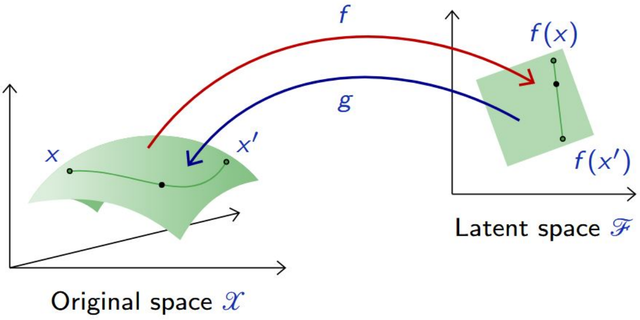
2. $g(\alpha x + (1 - \alpha)x')$



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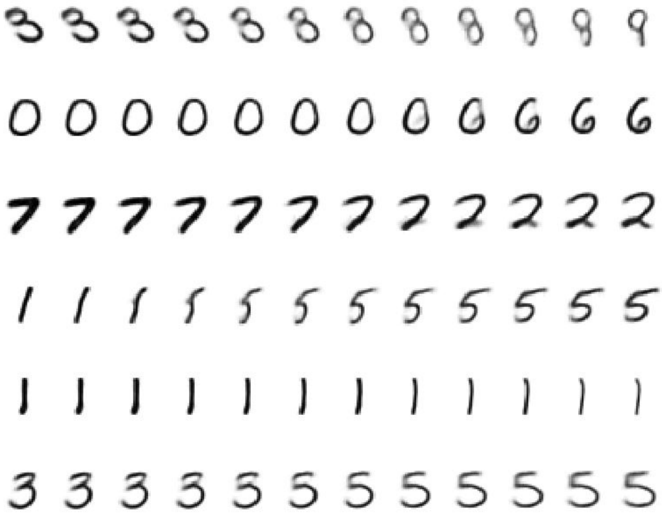

Generative Modeling by Autoencoder

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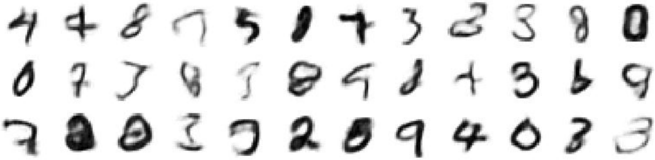
3. For instance, use a Gaussian density for modeling the latent space from the training data (estimate mean and a diagonal covariance matrix)

Generative Modeling by Autoencoder

Autoencoder sampling ($d = 8$)



Autoencoder sampling ($d = 16$)

Figure credits: Francois Fleuret

Generative Modeling by Autoencoder

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Generative Modeling by Autoencoder

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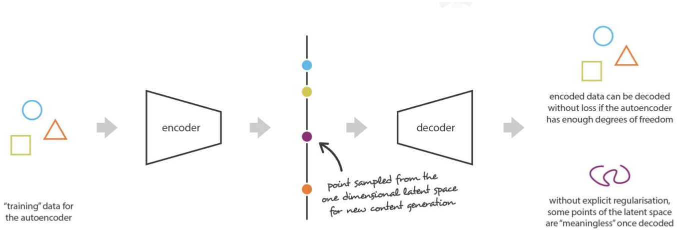
2. Because the density model is too simple
   - close points in latent space can give very different decoded data
   - some point of the latent space can give meaningless content once decoded

Generative Modeling by Autoencoder



A good model still needs to capture the empirical distribution on the data, although in a lower dimensional space