Deep Learning

15 Self-Attention & Transformers - II

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   - Computational cost for one forward pass: $\mathcal{O}(N^2.D^2)$

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   - No. of computations required for computing the dot products in self-attention layer $\mathcal{O}(N^2.D)$
4. Subsequent Neural Network layer has D inputs and D outputs $\rightarrow$ parameter $= \mathcal{O}(D^2)$ and computational cost of $\mathcal{O}(N.D^2)$

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4. We need a way to inject the order information

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3. Instead, add them $\tilde{x_n} = x_n + r_n$

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    - Skip connections retain the r_n across the layers

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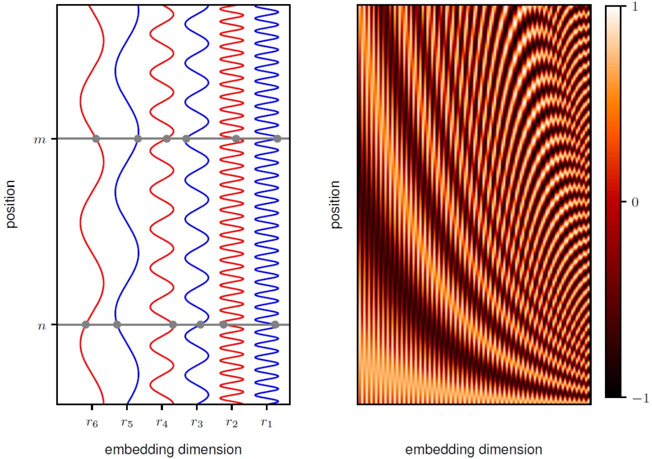
$$r_{ni} = \begin{cases} \sin\left(\frac{n}{L^{i/D}}\right), & \text{if } i \text{ is even,} \\ \cos\left(\frac{n}{L^{(i-1)/D}}\right), & \text{if } i \text{ is odd.} \end{cases}$$

The Bishop's book

Positional Encoding



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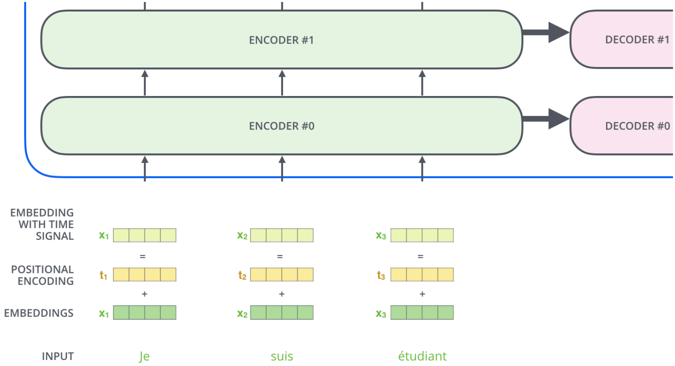
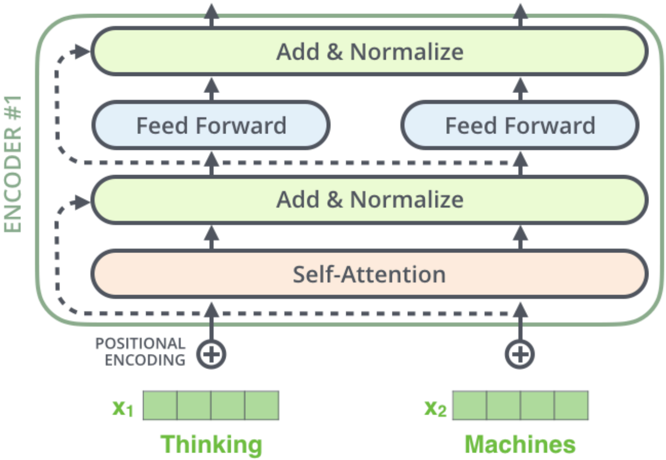


Credits: Jay Alammar

Residuals in the Encoder



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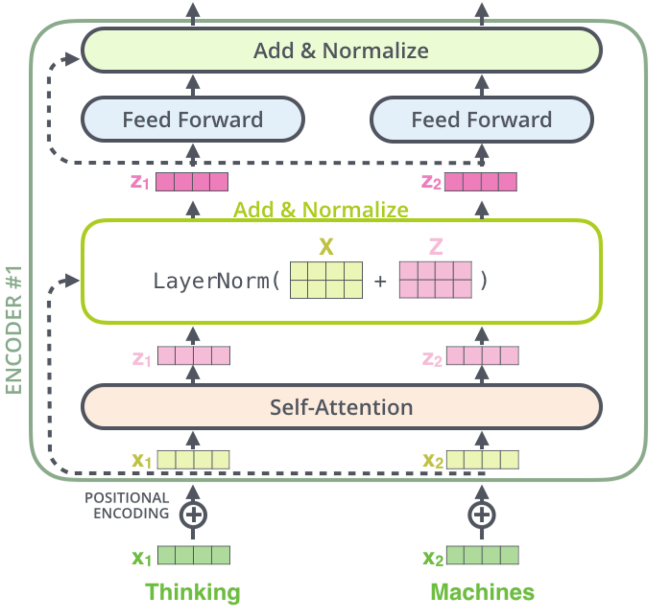
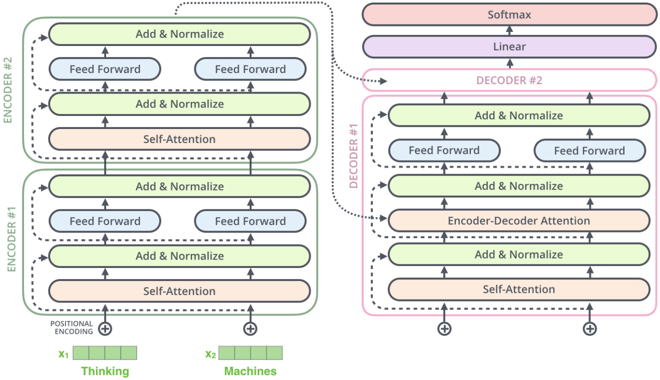


Tranformer-Decoder



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3. Encoder-decoder attention layer borrows the queries from the layer below it