Deep Learning

07 Cross-Entropy Loss

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Classification

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3. In other words, we don't prefer MSE loss for learning

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4. Loss function should compare $\mathbf{p}$ and $\mathbf{q}$

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3. Hence, the information can be calculated as $I(x) = -log_2(P(x))$
4. This is also the number of bits required to encode x

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2. Expected amount of information in an event drawn from that distribution $H(X) = \mathbb{E}_{x \sim p}[I(x)]$

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3. $H(p) = -\sum_i p_i \cdot log_2(p_i)$
4. Skewed distribution has less entropy, uniform/balanced distribution has more entropy

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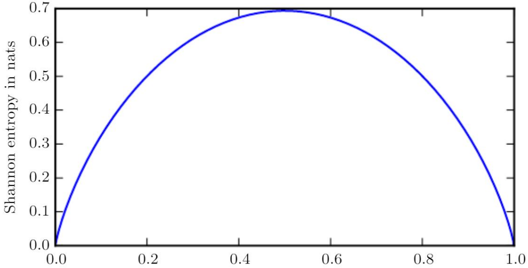

Entropy for a binary random variable

Figure credits Goodfellow et al. 2016

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4. Cross-entropy between a distribution and itself $(H(p,q))$ gives the entropy of the distribution $H(p)$

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2. $H(p,q) = H(p) + KL(p||q)$ where $KL(p||q) = \sum p_i \cdot log\left(\frac{p_i}{q_i}\right)$

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3. Target distribution (or, groundtruth) is one-hot encoding p, and model predicts a distribution q

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   - make them probabilities (i.e. sum to 1)

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2. $(\alpha_1, \alpha_2, \ldots, \alpha_C) \rightarrow (q_1, q_2, \ldots, q_C)$

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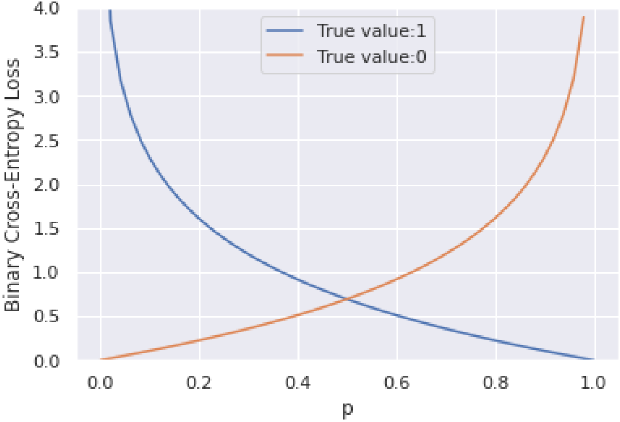
   - large if the model assigns smaller probability for the groundtruth class $(q_c \approx 0)$

BCE



True value:1
True value:0
Binary Cross-Entropy Loss
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.0
0.2
0.4
0.6
0.8
1.0
p