# Deep Learning 

## 13 Word Embeddings

Dr. Konda Reddy Mopuri<br>Dept. of AI, IIT Hyderabad Jan-May 2024

## Why Word Embeddings?



IITH has been consistently ranked in the top 10 institutes in India for Engineering according to NIRF making it one of the most coveted schools for science and technology in the country.

## Why Word Embeddings?



IITH has been consistently ranked in the to 10 institutes in India for Engineering according to NIRF making it one of the most coveted schools for science and technology in the country.

## Terminology

- Corpus: a collection of authentic text organized into dataset


## Terminology

- Corpus: a collection of authentic text organized into dataset
- Vocabulary (V): Set of allowed words


## Terminology

- Corpus: a collection of authentic text organized into dataset
- Vocabulary (V): Set of allowed words
- Target: Representation for every word in V


## One-hot Encoding

- Representation using discrete symbols


## One-hot Encoding

- Representation using discrete symbols
- $|V|$ words encoded as binary vectors of length $|V|$

Dictionary
Word Representation

A
Bus


Cat

| 0 | 0 | 1 | $\cdots \cdots \cdots$ | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- |

Tide

| 0 | 0 | 0 | $\ldots \ldots \ldots$ | 1 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- |

Zone

| 0 | 0 | 0 | $\ldots \ldots \ldots$ | 0 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- |

## One-hot encoding: Drawbacks

(1) Space inefficient (e.g. 13M words in Google 1T corpus)

## One-hot encoding: Drawbacks

(1) Space inefficient (e.g. 13M words in Google 1T corpus)
(2) No notion of similarity (or, distance) between words

## One-hot encoding: Drawbacks

(1) Space inefficient (e.g. 13M words in Google 1T corpus)
(2) No notion of similarity (or, distance) between words

- 'Dog' is as close to 'Cat' as it is to 'Machine'


## Notion of Meaning for words

(1) What is a good notion of meaning for a word?

## Notion of Meaning for words

(1) What is a good notion of meaning for a word?
(2) How do we, humans, know the meaning of a word?

## Notion of Meaning for words

(1) What does silla mean?

## Notion of Meaning for words

(1) Let's see how this word is used in different contexts

1. The silla is by the window, offering a nice view of the garden.
2. Can you pass me that silla so I can join the conversation?
3. After the event, please stack the sillas neatly against the wall.
4. I found a comfortable silla in the corner and settled down to relax.

## Notion of Meaning for words

(1) Does this context help you understand the word silla?

1. The silla is by the window, offering a nice view of the garden.
2. Can you pass me that silla so I can join the conversation?
3. After the event, please stack the sillas neatly against the wall.
4. I found a comfortable silla in the corner and settled down to relax.

## Notion of Meaning for words

(1) Does this context help you understand the word silla?
(2) \{ positioned near a window or against a wall or in the corner, used for conversing/events, can be used to relax \}

1. The silla is by the window, offering a nice view of the garden.
2. Can you pass me that silla so I can join the conversation?
3. After the event, please stack the sillas neatly against the wall.
4. I found a comfortable silla in the corner and settled down to relax.

## Notion of Meaning for words

(1) How did we do that?

## Notion of Meaning for words

(1) How did we do that?
(2) "We searched for other words that can be used in the same contexts, found some, and made a conclusion that silla has to mean similar to those words."

## Notion of Meaning for words

(1) Distributional Hypothesis: Words that frequently appear in similar contexts have a similar meaning

## Distributed Representations

(1) Representation/meaning of a word should consider its context in the corpus

## Distributed Representations

(1) Representation/meaning of a word should consider its context in the corpus
(2) Use many contexts of a word to build up a representation for it

## Distributed Representations

(1) Co-occurrence matrix is a way to can capture this!

- size: (\#words $\times$ \#words)
- rows: words (m), cols: context (n)
- words and context can be of same or different size


## Distributed Representations

(1) Co-occurrence matrix is a way to can capture this!

- size: (\#words $\times$ \#words)
- rows: words (m), cols: context (n)
- words and context can be of same or different size
(2) Context can be defined as a 'h' word neighborhood


## Distributed Representations

(1) Co-occurrence matrix is a way to can capture this!

- size: (\#words $\times$ \#words)
- rows: words (m), cols: context (n)
- words and context can be of same or different size
(2) Context can be defined as a ' $h$ ' word neighborhood
(3) Each row (column): vectorial representation of the word (context)


## Co-occurrence matrix

$$
X=\begin{gathered}
\text { I } \\
\text { I } \\
\text { like } \\
\text { enjoy } \\
\text { dearning } \\
\text { NLP } \\
\text { flying }
\end{gathered}\left[\begin{array}{cccccccc}
\text { like } & \text { enjoy } & \text { deep } & \text { learning } & \text { NLP } & \text { flying } & . \\
0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 0
\end{array}\right]
$$

## Co-occurrence matrix

(1) Very sparse

## Co-occurrence matrix

(1) Very sparse
(2) Very high-dimensional (grows with the vocabulary size)

## Co-occurrence matrix

(1) Very sparse
(2) Very high-dimensional (grows with the vocabulary size)
(3) Solution:Dimensionality reduction (SVD)!

## SVD on the Co-occurrence matrix

(1) $X=U \Sigma V^{T}$

## SVD on the Co-occurrence matrix

(1) $X=U \Sigma V^{T}$
(2) $[X]_{m \times n}=$

$$
\left[\begin{array}{ccc}
\uparrow & \ldots & \uparrow \\
u_{1} & \ldots & u_{k} \\
\downarrow & \cdots & \downarrow
\end{array}\right]_{m \times k} \cdot\left[\begin{array}{ccc}
\sigma_{1} & & \\
& \ddots & \\
& & \sigma_{k}
\end{array}\right]_{k \times k} \cdot\left[\begin{array}{ccc}
\leftarrow & v_{1}^{T} & \rightarrow \\
& \vdots & \\
\leftarrow & v_{k}^{T} & \rightarrow
\end{array}\right]_{k \times n}
$$

## SVD on the Co-occurrence matrix

(1) $X=U \Sigma V^{T}$
(2) $[X]_{m \times n}=$

$$
\left[\begin{array}{ccc}
\uparrow & \ldots & \uparrow \\
u_{1} & \cdots & u_{k} \\
\downarrow & \cdots & \downarrow
\end{array}\right]_{m \times k} \cdot\left[\begin{array}{ccc}
\sigma_{1} & & \\
& \ddots & \\
& & \sigma_{k}
\end{array}\right]_{k \times k} \cdot\left[\begin{array}{ccc}
\leftarrow & v_{1}^{T} & \rightarrow \\
& \vdots & \\
\leftarrow & v_{k}^{T} & \rightarrow
\end{array}\right]_{k \times n}
$$

(3) $X=\sigma_{1} u_{1} v_{1}^{T}+\sigma_{2} u_{2} v_{2}^{T}+\ldots+\sigma_{k} u_{k} v_{k}^{T}$
(4) $\hat{X}=\sum_{i=1}^{d<k} \sigma_{i} u_{i} v_{i}^{T}$ is a $d$-rank approximation of $X$

## SVD on the Co-occurrence matrix

(1) Before the SVD, representations were the rows of $X$

## SVD on the Co-occurrence matrix

(1) Before the SVD, representations were the rows of $X$
(2) How do we reduce the representation size with SVD ?

## SVD on the Co-occurrence matrix

(1) Before the SVD, representations were the rows of $X$
(2) How do we reduce the representation size with SVD ?
(3) $W_{\text {word }}=U_{m \times k} \cdot \Sigma_{k \times k}$

## SVD on the Co-occurrence matrix

(1) $W_{\text {word }} \in \mathbb{R}^{m \times k}(k \ll|V|=m)$ are considered the representation of the words

## SVD on the Co-occurrence matrix

(1) $W_{\text {word }} \in \mathbb{R}^{m \times k}(k \ll|V|=m)$ are considered the representation of the words
(2) Lesser dimensions but the same similarities! (one may verify that $X X^{T}=\hat{X} \hat{X}^{T}$ )

## SVD on the Co-occurrence matrix

(1) $W_{\text {word }} \in \mathbb{R}^{m \times k}(k \ll|V|=m)$ are considered the representation of the words
(2) Lesser dimensions but the same similarities! (one may verify that $\left.X X^{T}=\hat{X} \hat{X}^{T}\right)$
(3) $W_{\text {context }}=V \in \mathbb{R}^{n \times k}$ are taken as the representations for the context words

## A bit more clever things...

(1) Entries in the occurrence matrix can be weighted (HAL model)

## A bit more clever things...

(1) Entries in the occurrence matrix can be weighted (HAL model)
(2) Better associations can be used (PPMI)

## A bit more clever things...

(1) Entries in the occurrence matrix can be weighted (HAL model)
(2) Better associations can be used (PPMI)
(3) ....

# Count-based vs prediction-based models 

(1) Techniques we have seen so far rely on the counts (or, co-occurrence of words)

## Count-based vs prediction-based models

(1) Techniques we have seen so far rely on the counts (or, co-occurrence of words)
(2) Next, we see prediction based models for word embeddings

## Word2Vec

(1) T Mikolov et al. (2013)

## Word2Vec

(1) T Mikolov et al. (2013)
(2) Two versions: Predict words from the contexts (or contexts from words)

## Word2Vec

(1) T Mikolov et al. (2013)
(2) Two versions: Predict words from the contexts (or contexts from words)
(3) Continuous Bag of Words (CBoW) and Skip-gram

## Word2Vec

Input Projection Output


CBOW

Input Projection Output


Skip-gram

## Word Embeddings: Skip-gram



## Skip-gram

## Word Embeddings: Skip-gram



Input layer

## Word Embeddings: Skip-gram

(1) Start: huge corpus and random initialization of the word embeddings

## Word Embeddings: Skip-gram

(1) Start: huge corpus and random initialization of the word embeddings
(2) Process the text with a sliding window (one word at a time)

## Word Embeddings: Skip-gram

(1) Start: huge corpus and random initialization of the word embeddings
(2) Process the text with a sliding window (one word at a time)
(1) At each step, there is a central word and context words (other words in the window)

## Word Embeddings: Skip-gram

(1) Start: huge corpus and random initialization of the word embeddings
(2) Process the text with a sliding window (one word at a time)
(1) At each step, there is a central word and context words (other words in the window)
(2) Given the central word, compute the probabilities for the context words

## Word Embeddings: Skip-gram

(1) Start: huge corpus and random initialization of the word embeddings
(2) Process the text with a sliding window (one word at a time)
(1) At each step, there is a central word and context words (other words in the window)
(2) Given the central word, compute the probabilities for the context words
(3) Modify the word embeddings to increase these probabilities

## Word Embeddings: Skip-gram

$$
P\left(w_{t-2} \mid w_{t}\right) P\left(w_{t-1} \mid w_{t}\right) P\left(w_{t+1} \mid w_{t}\right) P\left(w_{t+2} \mid w_{t}\right)
$$

... I saw a cute grey cat playing in the garden ...

$$
\begin{array}{lllll}
w_{t-2} & w_{t-1} & w_{t} & w_{t+1} & w_{t+2}
\end{array}
$$

context central context
words word words

Figure from Lena Voita

## Word Embeddings: Skip-gram



Figure from Lena Voita

## Word Embeddings: Skip-gram

$$
P\left(w_{t-2} \mid w_{t}\right) P\left(w_{t-1} \mid w_{t}\right) P\left(w_{t+1} \mid w_{t}\right) P\left(w_{t+2} \mid w_{t}\right)
$$

... I saw a cute grey cat playing in the garden ...

$$
w_{t-2} \quad w_{t-1} \quad w_{t} \quad w_{t+1} \quad w_{t+2}
$$

$$
\begin{array}{ccc}
\text { context } & \text { central } & \text { context } \\
\text { words } & \text { word } & \text { words }
\end{array}
$$

Figure from Lena Voita

## Word Embeddings: Skip-gram

$$
P\left(w_{t-2} \mid w_{t}\right) P\left(w_{t-1} \mid w_{t}\right) P\left(w_{t+1} \mid w_{t}\right) P\left(w_{t+2} \mid w_{t}\right)
$$

... I saw a cute grey cat playing in the garden

$$
w_{t-2} \quad w_{t-1} \quad w_{t} \quad w_{t+1} \quad w_{t+2}
$$

$$
\begin{array}{ccc}
\text { context } & \text { central } & \text { context } \\
\text { words } & \text { word } & \text { words }
\end{array}
$$

Figure from Lena Voita

## Word Embeddings: Skip-gram



Figure from Lena Voita

## Word Embeddings: Skip-gram

- For each position in $t=1,2, \ldots T$ in the corpus, Skip-gram predicts the context words in $m$-sized window ( $\theta$ is the variables to be optimized)

$$
\text { Likelihood } L(\theta)=\prod_{t=1}^{T} \prod_{-m \leq j \leq m, j \neq 0} P\left(w_{t+j} \mid w_{t}, \theta\right)
$$

## Word Embeddings: Skip-gram

- The loss is mean NLL

$$
\text { Loss } J(\theta)=-\frac{1}{T} \prod_{t=1}^{T} \prod_{-m \leq j \leq m, j \neq 0} \log P\left(w_{t+j} \mid w_{t}, \theta\right)
$$

## Word Embeddings: Skip-gram

- What are the parameters $(\theta)$ to be learned?


Figure from Lena Voita

## Word Embeddings: Skip-gram

- How to compute $P\left(w_{t+j} \mid w_{t}, \theta\right)$ ?


Figure from Lena Voita

## Word Embeddings: Skip-gram

$P\left(u_{I} \mid v_{a}\right) P\left(u_{\text {saw }} \mid v_{a}\right) P\left(u_{\text {cute }} \mid v_{a}\right) P\left(u_{\text {grey }} \mid v_{a}\right)$
... I saw a cute grey cat playing in the garden
$w_{t-2} \quad w_{t-1} \quad w_{t} \quad w_{t+1} \quad w_{t+2}$

$v$

$u$

Figure from Lena Voita

## Word Embeddings: Skip-gram

$$
P\left(u_{\text {saw }} \mid v_{\text {cute }}\right) P\left(u_{a} \mid v_{\text {cute }}\right) P\left(u_{\text {grey }} \mid v_{\text {cute }}\right) P\left(u_{\text {cat }} \mid v_{\text {cute }}\right)
$$

... I saw a cute grey cat playing in the garden

$$
w_{t-2} \quad w_{t-1} \quad w_{t} \quad w_{t+1} \quad w_{t+2}
$$



$u$

Figure from Lena Voita

## Word Embeddings: Skip-gram



Figure from Lena Voita

## Word Embeddings: Skip-gram


$w_{t-2} \quad w_{t-1} \quad w_{t} \quad w_{t+1} \quad w_{t+2}$

$v$

$u$

Figure from Lena Voita

## Word Embeddings: Skip-gram

$$
P\left(u_{\text {grey }} \mid v_{\text {playing }}\right) P\left(u_{\text {cat }} \mid v_{\text {playing }}\right) P\left(u_{\text {in }} \mid v_{\text {playing }}\right) P\left(u_{\text {the }} \mid v_{\text {playing }}\right)
$$

... I saw a cute grey cat playing in the garden

$$
\begin{array}{lllll}
w_{t-2} & w_{t-1} & w_{t} & w_{t+1} & w_{t+2}
\end{array}
$$

$\square$


Figure from Lena Voita

## Word Embeddings: Skip-gram



Figure from Lena Voita

## Word Embeddings: Skip-gram

- Train using Gradient Descent


## Word Embeddings: Skip-gram

- Train using Gradient Descent
- For one word at a time, i.e., (a center word, one of the context words)


## Word Embeddings: Skip-gram

- Train using Gradient Descent
- For one word at a time, i.e., (a center word, one of the context words)
- $J_{t, j}(\theta)=-\log P($ cute|cat $)=-\log \frac{\exp u_{\text {cut }}^{T} v_{\text {cat }}}{\sum_{w \in V_{o c}} \exp u_{w}^{T} v_{c a t}}=$ $-u_{c u t e}^{T} v_{c a t}+\log \sum_{w \in V o c} \exp u_{w}^{T} v_{c a t}$


## Word Embeddings: Skip-gram

1. Take dot product of $v_{\text {cat }}$ with all $u$
2. exp
3. sum all

4. get loss (for this one step)

$$
J_{t, j}(\theta)=-\underbrace{u_{c u t e}^{T} v_{c a t}}_{1}+\log \underbrace{\sum_{w \in V} \exp \left(u_{w}^{T} v_{c a t}\right)}_{1}
$$

$$
v_{c a t}:=v_{c a t}-\alpha \frac{\partial J_{t, j}(\theta)}{\partial v_{c a t}}
$$

$$
u_{w}:=u_{w}-\alpha \frac{\partial J_{t, j}(\theta)}{\partial u_{w}} \forall w \in \mathrm{~V}
$$

Figure from Lena Voita

## Word Embeddings: Skip-gram

- Training is slow (for each central word, all the context words need to be updated)


## Word Embeddings: Skip-gram

- Training is slow (for each central word, all the context words need to be updated)
- Negative sampling: not all the context words are considered, but a random sample of them


## Word Embeddings: Skip-gram

- Training is slow (for each central word, all the context words need to be updated)
- Negative sampling: not all the context words are considered, but a random sample of them
- Training over a large corpus leads to sufficient updates for each vector


## Word Embeddings: Skip-gram

Dot product of $v_{\text {cat }}$ :

- with $u_{\text {cute }}$-increase,
- with all other $u$ - decrease


Parameters to be updated:

- $v_{\text {cat }}$
- $u_{w}$ for all $w$ in $|\mathrm{V}|+1$ vectors the vocabulary

Dot product of $v_{\text {cat }}$ :

- with $u_{\text {cute }}$-increase,
- with a subset of other $u$-decrease

Negative samples: randomly
selected K words


Parameters to be updated:

- $v_{\text {cat }}$
- $u_{\text {cute }}$ and $u_{w}$ for $w K+2$ vectors in K negative examples

Figure from Lena Voita

## Word Embeddings: Skip-gram



Can be viewed as a Neural Network

## Word Embeddings: Skip-gram



Can be viewed as a Neural Network

## Word Embeddings: Skip-gram



Can be viewed as a Neural Network

## Word Embeddings: Skip-gram



Can be viewed as a Neural Network

## Word Embeddings: Skip-gram

(1) $W_{N \times m}$ is the $W_{\text {word }}$ (used for representing the words)

## Word Embeddings: Skip-gram

(1) $W_{N \times m}$ is the $W_{\text {word }}$ (used for representing the words)
(2) $W_{m \times N}^{\prime}$ is the $W_{\text {context }}$ (may be ignored after the training)

## Word Embeddings: Skip-gram

(1) $W_{N \times m}$ is the $W_{\text {word }}$ (used for representing the words)
(2) $W_{m \times N}^{\prime}$ is the $W_{\text {context }}$ (may be ignored after the training)
(3) Some showed averaging word and context vectors may be more beneficial

## Bag of Words (BoW)

(1) Bag of Words: Collection and frequency of words


## CBoW

(1) Considers the embeddings of ' $h$ ' words before and ' $h$ ' words after the target word

## CBoW

(1) Considers the embeddings of ' $h$ ' words before and ' $h$ ' words after the target word
(2) Adds them (order is lost) for predicting the target word

cBOW

## CBoW

The dog slept on couch


## CBoW

(1) Size of the vocabulary $=m$

## CBoW

(1) Size of the vocabulary $=m$
(2) Dimension of the embeddings $=N$

## Word Embeddings: CBoW

(1) Input layer $W_{N \times m}$ (embeddings for the context words) projects the context (sum of 1 -hot vectors of all the context vectors) into N -dim space
context

$$
\left(W_{N \times m}\right)\left(\begin{array}{l}
c_{m \times 1}
\end{array}\right)
$$

## Word Embeddings: CBoW

(1) Input layer $W_{N \times m}$ (embeddings for the context words) projects the context (sum of 1 -hot vectors of all the context vectors) into N -dim space
(2) Representations of all the $(2 h)$ words in the context are summed (result is an $N$-dim context vector)
context

$$
\left(w_{N \times m}\right)\left(c_{m \times 1}\right)
$$

## Word Embeddings: CBoW

(1) Input layer $W_{N \times m}$ (embeddings for the context words) projects the context (sum of 1 -hot vectors of all the context vectors) into N -dim space
context

$$
\left(W_{N \times m}\right)\left(C_{m \times 1}\right)\left(E_{N \times 1}\right)
$$

## Word Embeddings: CBoW

(1) Input layer $W_{N \times m}$ (embeddings for the context words) projects the context (sum of 1 -hot vectors of all the context vectors) into N -dim space
(2) Representations of all the $(2 h)$ words in the context are summed (result is an $N$-dim context vector)
context

$$
(-)[-][-1]
$$

## Word Embeddings: CBoW

(1) Next layer has a weight matrix $W_{m \times N}^{\prime}$ (embeddings for the center words)

## Word Embeddings: CBoW

(1) Next layer has a weight matrix $W_{m \times N}^{\prime}$ (embeddings for the center words)
(2) Projects the accumulated embeddings onto the vocabulary


## Word Embeddings: CBoW

(1) Next layer has a weight matrix $W_{m \times N}^{\prime}$ (embeddings for the center words)
(2) Projects the accumulated embeddings onto the vocabulary

$$
\left(w_{N \times m}\right)\left[c_{m \times 1}\right) \Rightarrow\left(w_{m \times N}^{\prime}\right)\left(E_{N \times 1}\right) \Rightarrow\left(c_{\substack{m \times 1}}\right]_{\substack{\text { Scoress ofr } r \text { wway } \\ \text { clossficiotion }}}
$$

## Word Embeddings: CBoW

(1) $m$ - way classification $\rightarrow$ (after a softmax) maximizes the probability for the target word


## Word Embeddings: CBoW

(1) $W_{N \times m}$ is the $W_{\text {context }}$

## Word Embeddings: CBoW

(1) $W_{N \times m}$ is the $W_{\text {context }}$
(2) $W_{m \times N}^{\prime}$ is the $W_{\text {words }}$

## Glove

# (1) Glove - Global Vectors 

## Glove

(1) Glove - Global Vectors
(2) Combines the score-based and predict-based approaches

## Glove

(1) $X_{i j}$ in the co-occurrence matrix encodes the global info. about words $i$ and $j$

## Glove

(1) $X_{i j}$ in the co-occurrence matrix encodes the global info. about words $i$ and $j$

- $p(j / i)=\frac{X_{i j}}{X_{i}}$
(2) Glove attempts to learn representations that are faithful to the co-occurrence info.


## Glove

(1) Glove attempts to learn representations that are faithful to the co-occurrence info.

## Glove

(1) Glove attempts to learn representations that are faithful to the co-occurrence info.
(2) Try to enforce $v_{i}^{T} c_{j}=\log P(j / i)=\log X_{i j}-\log X_{i}$

- $v_{i}$ - central representation of word $i, c_{j}$ - context representation of word $j$


## Glove

(1) Glove attempts to learn representations that are faithful to the co-occurrence info.
(2) Try to enforce $v_{i}^{T} c_{j}=\log P(j / i)=\log X_{i j}-\log X_{i}$

- $v_{i}$ - central representation of word $i, c_{j}$ - context representation of word $j$
(3) Similarly, $v_{j}^{T} c_{i}=\log P(i / j)=\log X_{i j}-\log X_{j}$ (aim is to learn such embeddings $v_{i}$ and $c_{i}$ )


## Glove

(1) To realize the exchange symmetry of $v_{i}^{T} c_{j}=\log P(j / i)=\log X_{i j}-\log X_{i}$

## Glove

(1) To realize the exchange symmetry of $v_{i}^{T} c_{j}=\log P(j / i)=\log X_{i j}-\log X_{i}$

- we may capture the $\log X_{i}$ as a bias $b_{i}$ of the word $w_{i}$


## Glove

(1) To realize the exchange symmetry of $v_{i}^{T} c_{j}=\log P(j / i)=\log X_{i j}-\log X_{i}$

- we may capture the $\log X_{i}$ as a bias $b_{i}$ of the word $w_{i}$
- And, an additional term $\tilde{b_{j}}$


## Glove

(1) To realize the exchange symmetry of $v_{i}^{T} c_{j}=\log P(j / i)=\log X_{i j}-\log X_{i}$

- we may capture the $\log X_{i}$ as a bias $b_{i}$ of the word $w_{i}$
- And, an additional term $\tilde{b_{j}}$
(2) $v_{i}^{T} c_{j}+b_{i}+\tilde{b_{j}}=\log X_{i j}$


## Glove

(1) To realize the exchange symmetry of $v_{i}^{T} c_{j}=\log P(j / i)=\log X_{i j}-\log X_{i}$

- we may capture the $\log X_{i}$ as a bias $b_{i}$ of the word $w_{i}$
- And, an additional term $\tilde{b}_{j}$
(2) $v_{i}^{T} c_{j}+b_{i}+\tilde{b_{j}}=\log X_{i j}$
(3) Since $\log X_{i}$ and $\log X_{j}$ depend on the words $i$ and $j$, they can be considered as the word specific biases (learnable)


## Glove

(1) Learning objective becomes

$$
\underset{v_{i}, c_{j}, b_{i}, \tilde{b_{j}}}{\operatorname{argmin}} J()=\sum_{i, j}\left(v_{i}^{T} c_{j}+b_{i}+\tilde{b}_{j}-\log X_{i j}\right)^{2}
$$

## Glove

(1) Learning objective becomes
$\underset{\tilde{\sim}}{\operatorname{argmin}} J()=\sum_{i, j}\left(v_{i}^{T} c_{j}+b_{i}+\tilde{b}_{j}-\log X_{i j}\right)^{2}$ $v_{i}, c_{j}, b_{i}, \tilde{b}_{j}$

## Glove

(1) Learning objective becomes
$\underset{\tilde{\sim}}{\operatorname{argmin}} J()=\sum_{i, j}\left(v_{i}^{T} c_{j}+b_{i}+\tilde{b}_{j}-\log X_{i j}\right)^{2}$ $v_{i}, c_{j}, b_{i}, \tilde{b}_{j}$
(2) Much of the entries in the co-occurrence matrix are zeros (noisy or less informative)

## Glove

(1) Learning objective becomes
$\underset{\sim}{\operatorname{argmin}} J()=\sum_{i, j}\left(v_{i}^{T} c_{j}+b_{i}+\tilde{b}_{j}-\log X_{i j}\right)^{2}$ $v_{i}, c_{j}, b_{i}, \tilde{b}_{j}$
(2) Much of the entries in the co-occurrence matrix are zeros (noisy or less informative)
(3) Suggests to apply a weight

## Glove

$$
J(\theta)=\sum_{w, c \in V} f(\mathrm{~N}(\mathrm{w}, \mathrm{c})) \cdot\left(u_{c}^{T} v_{w}+b_{c}+\overline{b_{w}}-\log \mathrm{N}(\mathrm{w}, \mathrm{c})\right)^{2}
$$

Weighting function to:

- penalize rare events
- not to over-weight frequent events


Figure from Lena Voita

## Evaluating the embeddings

(1) Intrinsic - studying the internal properties (how well they capture the meaning: word similarity, analogy, etc.)

## Evaluating the embeddings

(1) Intrinsic - studying the internal properties (how well they capture the meaning: word similarity, analogy, etc.)
(2) Extrinsic - studying how they perform a task

## Analysing the embeddings

(1) Walking the semantic space

## Analysing the embeddings

(1) Walking the semantic space
(2) Structure - (form clusters) nearest neighbors have a similar meaning, Linear structure

## Glove

semantic: $\quad v($ king $)-v($ man $)+v($ woman $) \approx v($ queen $)$ syntactic: $v($ kings $)-v($ king $)+v($ queen $) \approx v($ queens $)$


Figure from Lena Voita

## Glove



Figure from Lena Voita

