Deep Learning

01 The Artificial Neuron (MP Neuron and Perceptron)

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Jan-May 2024

The Neuron

- About 100 billion neurons in human brain

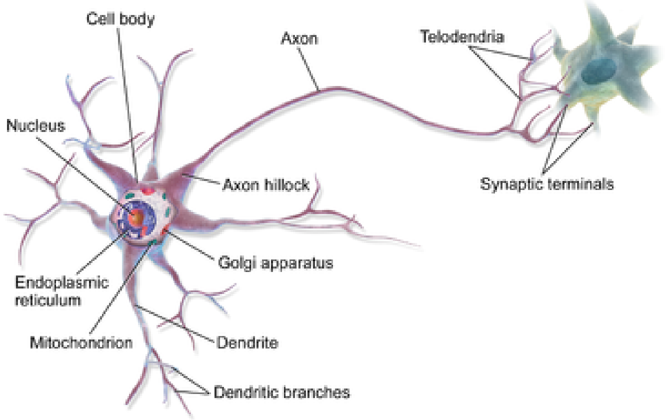
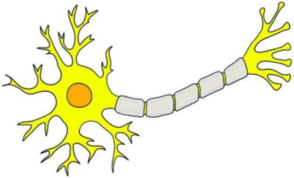


Figure credits: Wikipedia

The dilemma: To watch or not to watch?

Let's use our brain



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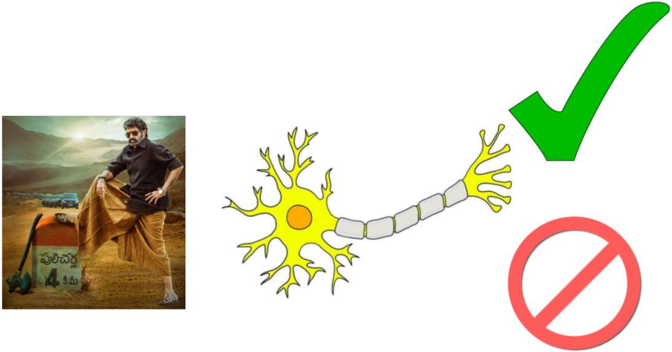
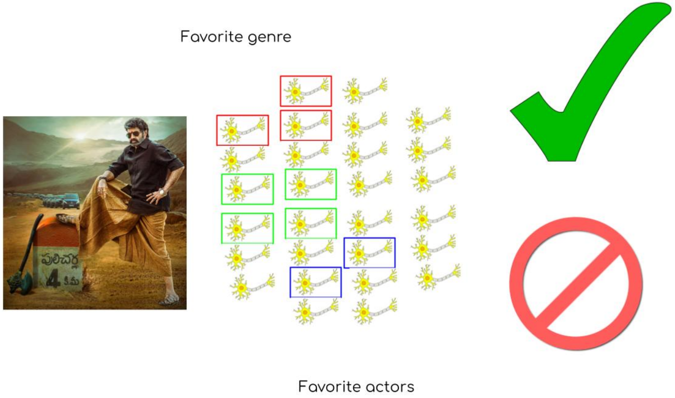
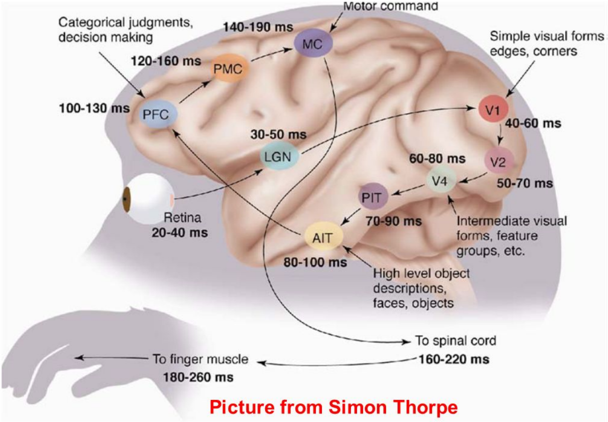


It’s a network of many neurons

There is a division of responsibilities



Neurons in the brain have a hierarchy



Picture from Simon Thorpe

Threshold Logic Unit

1. First Mathematical Model for a neuron

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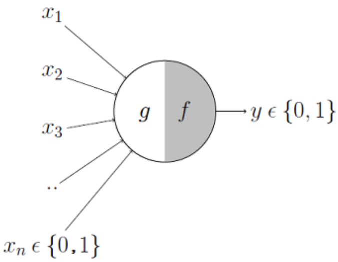

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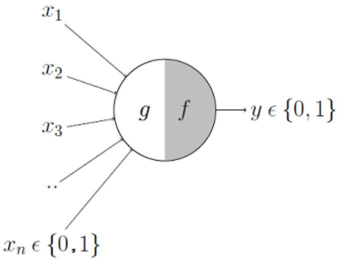

4. $$f(x) = \mathbb{1}(\sum_i x_i \geq \theta)$$

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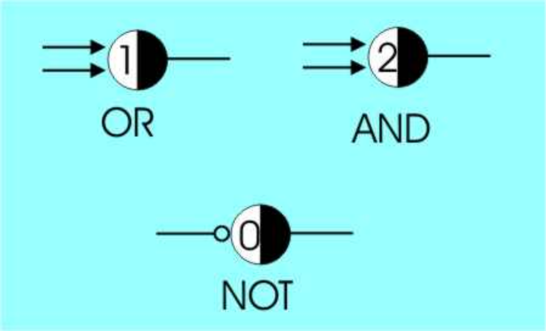
3. Counts the number of 'ON' signals on the excitatory inputs versus the inhibitory

Threshold Logic Unit



Example Boolean functions

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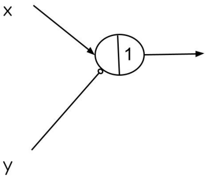

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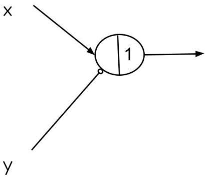
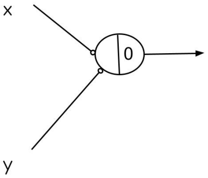


3. NOR



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2. No learning; heuristic approach

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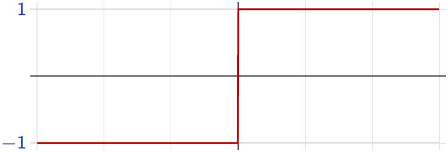
5. $$f(x) = \begin{cases} 1 & \text{when } \sum_i w_i x_i + b \geq 0 \\ 0 & \text{else} \end{cases}$$

Perceptron

1. For simplicity we consider +1 and -1 responses

$$\sigma(x) = \begin{cases} 1 & \text{when } x \geq 0 \\ -1 & \text{else} \end{cases}$$



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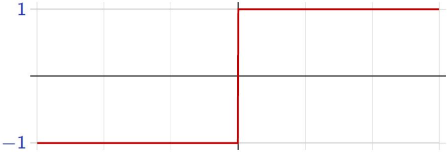

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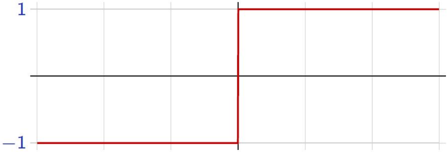

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3. $\mathbf{w}$ are referred to as weights and b as the bias

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4. Mechanism for learning the weights

Weights and Bias

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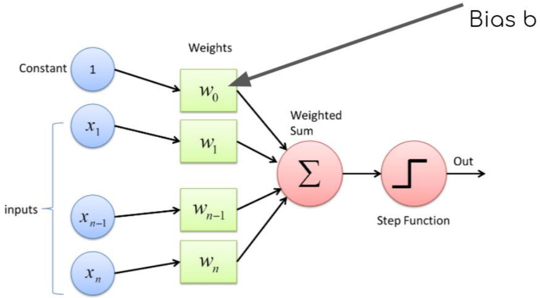
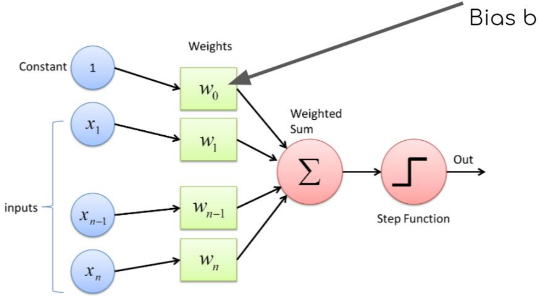


Figure credits: DeepAI

Weights and Bias

1. Why are the weights important?
2. Why is it called 'bias'? What does it capture?



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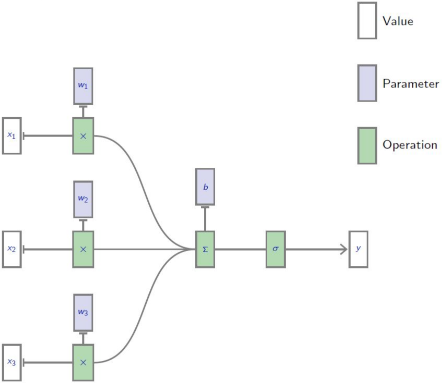

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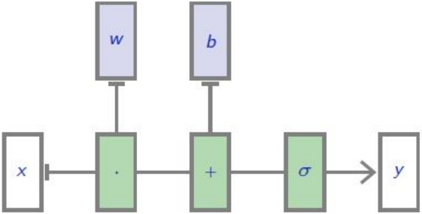

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   $k \leftarrow k + 1$

Perceptron Learning algorithm

భారతీయ సాంకేతిక విజ్ఞాన సంస్థ హైదరాబాద్
भारतीय प्रौद्योगिकी संस्थान हैदराबाद
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4. Note that the bias b is absorbed as a component of $\mathbf{w}$ and $\mathbf{x}$ is appended with 1 suitably

Perceptron Learning Algorithm

▸ Colab Notebook: Perceptron-learning

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3. Other algorithms maximize the margin from the boundary to the samples

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4. What if the data is not linearly separable?