Deep Learning

6 Backpropagation-2

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भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

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- Then wrt the parameters
$\frac{\partial \ell}{\partial w_{i,j}^{(l)}} = \frac{\partial \ell}{\partial s_i^{(l)}} x_j^{(l-1)}$ and $\frac{\partial \ell}{\partial b_i^{(l)}} = \frac{\partial \ell}{\partial s_i^{(l)}}$

Jocobian in Tensorial form

- $\psi : \mathcal{R}^N \rightarrow \mathcal{R}^M$ then $\left[\frac{\partial \psi}{\partial x}\right] = \begin{bmatrix} \frac{\partial \psi_1}{\partial x_1} & \cdots & \frac{\partial \psi_1}{\partial x_N} \\ \vdots & \ddots & \vdots \\ \frac{\partial \psi_M}{\partial x_1} & \cdots & \frac{\partial \psi_M}{\partial x_N} \end{bmatrix}$

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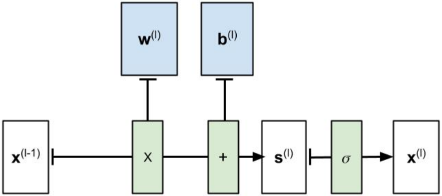
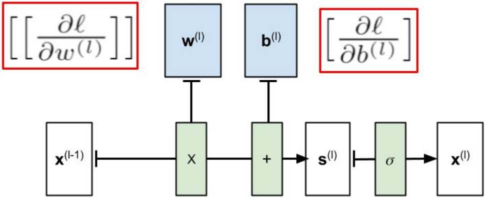
- $\psi : \mathcal{R}^{N \times M} \rightarrow \mathcal{R}$ then $\left[\left[\frac{\partial \psi}{\partial x}\right]\right] = \begin{bmatrix} \frac{\partial \psi}{\partial w_{1,1}} & \cdots & \frac{\partial \psi}{\partial w_{1,M}} \\ \vdots & \ddots & \vdots \\ \frac{\partial \psi}{\partial w_{N,1}} & \cdots & \frac{\partial \psi}{\partial w_{N,M}} \end{bmatrix}$

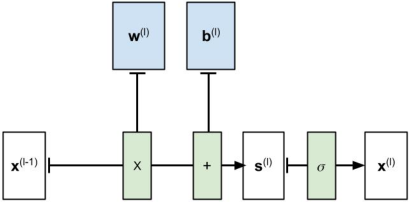
Forward Pass



Goal of Backward Pass



Begin from succeeding layer



$$\boxed{\frac{\partial \ell}{\partial x^{(l)}}}$$

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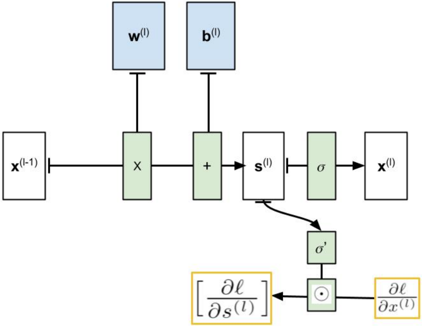

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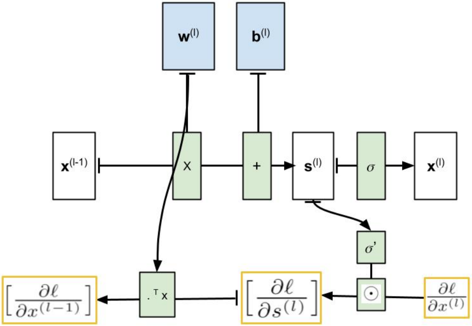

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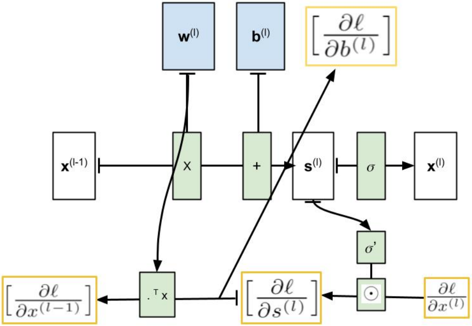

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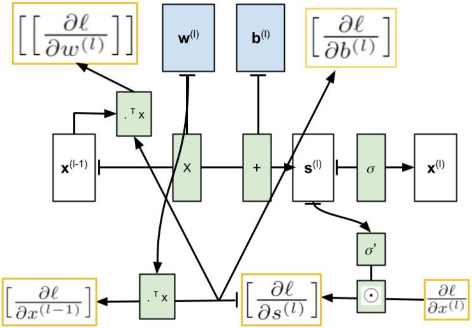

Update the parameters

- $W^{(l)} = W^{(l)} - \eta \left[\left[\frac{\partial \ell}{\partial w^{(l)}}\right]\right]$ and $\mathbf{b}^{(l)} = \mathbf{b}^{(l)} - \eta \left[\frac{\partial \ell}{\partial b^{(l)}}\right]$

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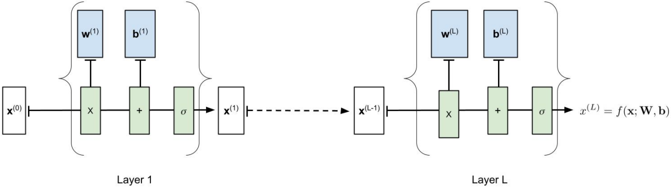
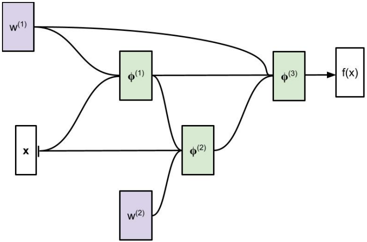
- In an untreated situation, BP Needs all the intermediate layer results to be in memory
- Takes twice the computations of forward pass

Beyond MLP

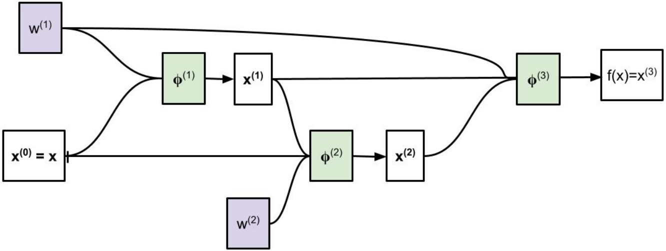
- We can generalize MLP





To an arbitrary Directed Acyclic Graph (DAG)

Forward pass in the computational graph



- $x^{(0)} = x$

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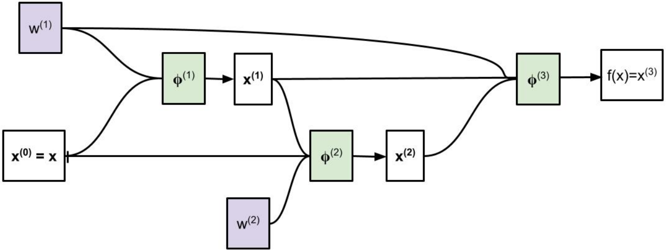

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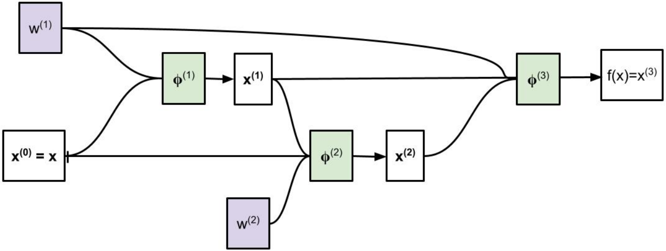

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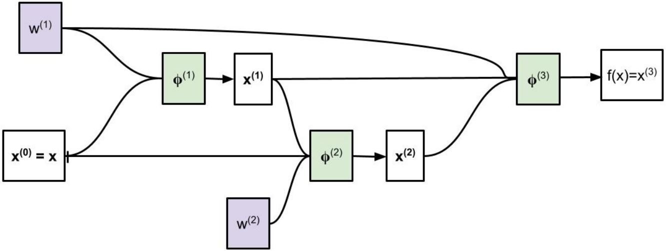

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- $x^{(2)} = \phi^{(2)}(x^{(0)}, x^{(1)}; w^{(2)})$
- $f(x) = x^{(3)} = \phi^{(3)}(x^{(1)}, x^{(2)}; w^{(1)})$

Notation: Jacobian of a general transformation

- $$\text{if } (a_1 \dots a_Q) = \phi(b_1 \dots b_R) \text{ then we use the notation} \tag{3}$$

$$\left[\frac{\partial a}{\partial b}\right] = J_\phi^T = \begin{bmatrix} \frac{\partial a_1}{\partial b_1} & \cdots & \frac{\partial a_Q}{\partial b_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial a_1}{\partial b_R} & \cdots & \frac{\partial a_Q}{\partial b_R} \end{bmatrix} \tag{4}$$

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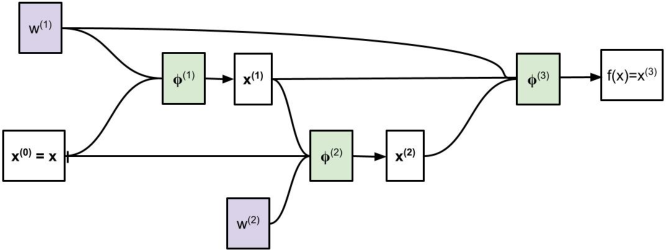
Backward pass



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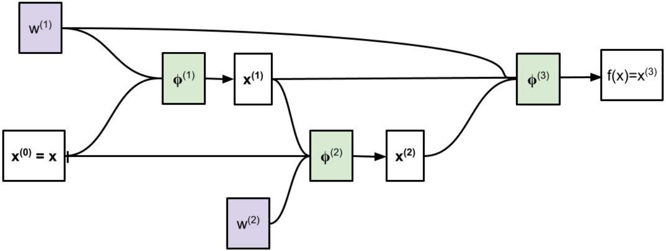

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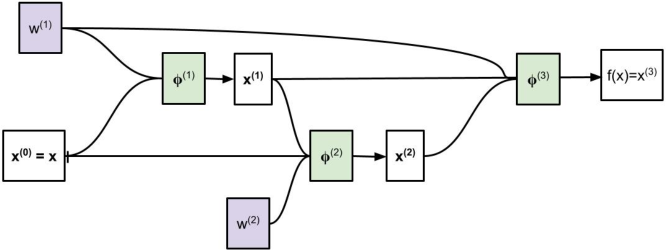

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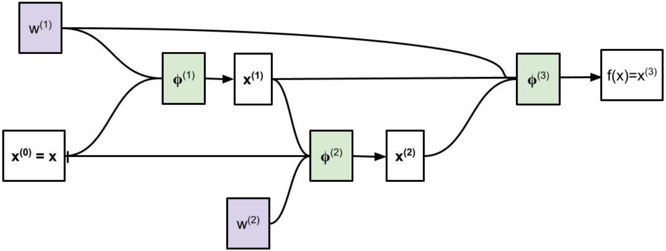

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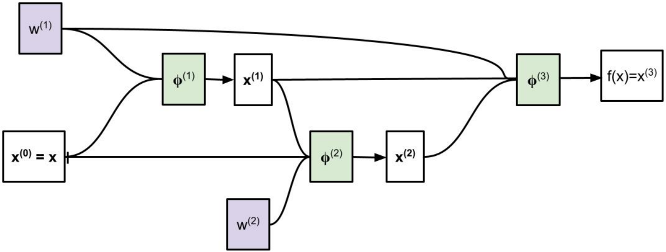

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- Default is False
- `requires_grad_()` function can be used to set to any value

Autograd

- `torch.autograd.grad(o/p,i/p)` returns gradients of outputs wrt the inputs

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- Accumulation is helpful (e.g. sum of losses, or sum over different mini-batches, etc.)

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- Used when gradient should not be propagated beyond a variable, or to update the leaf nodes in the graph

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- Specified with `create_graph = True`