Deep Learning

20 Generative Adversarial Network (GAN)

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1. Work by Ian Goodfellow et al. (NeurIPS 2014)

Goal

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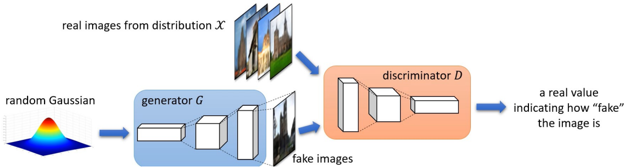
3. Draws samples that are similar to the train data (but not exactly them)

Method



Credit: Microsoft research blog

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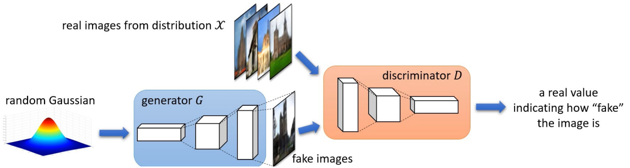

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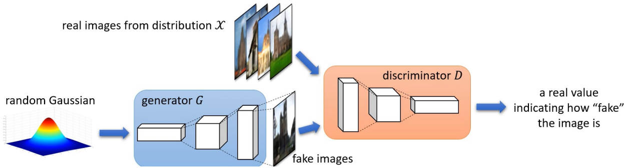

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3. Machinery to ensure $p_G \approx p_{\text{data}}$

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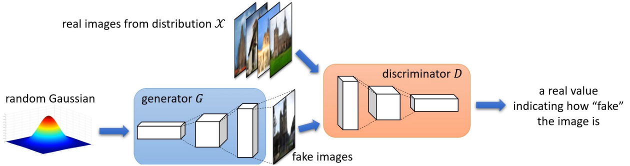

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1. Employ a classifier to differentiate between **real** samples $x \sim p_{\text{data}}$ (label 1) and **generated**(fake) ones $\hat{x} \sim p_G$ (label 0)

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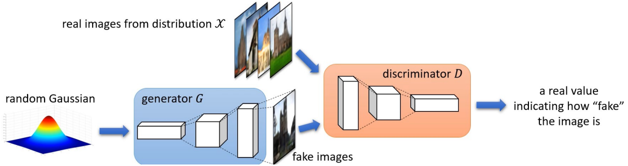

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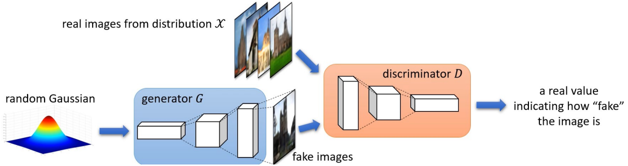

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3. Train the G such that D misclassifies generated samples $\hat{x}$ into class 1 (can't differentiate b/w $x \sim p_{\text{data}}$ and $\hat{x} \sim p_G$)

Training Objective

$$\text{min}_G \ \text{max}_D \left(\mathbb{E}_{x \sim p_{\text{data}}}[logD(x)] + \mathbb{E}_{z \sim p_z}[log(1 - D(G(z)))] \right)$$

1. minmax optimization (or, zero-sum game)

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3. Expectation in practice is average over a batch of samples

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5. Which would be ≈ 0 for a confident $D \rightarrow$ (no gradients to train G!)

Training Strategy

Algorithm 1 Minibatch stochastic gradient descent training of generative adversarial nets. The number of steps to apply to the discriminator, k, is a hyperparameter. We used $k = 1$, the least expensive option, in our experiments.

for number of training iterations **do**
  for k steps **do**
    • Sample minibatch of m noise samples $\{\boldsymbol{z}^{(1)}, \ldots, \boldsymbol{z}^{(m)}\}$ from noise prior $p_g(\boldsymbol{z})$.
    • Sample minibatch of m examples $\{\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(m)}\}$ from data generating distribution $p_{\text{data}}(\boldsymbol{x})$.
    • Update the discriminator by ascending its stochastic gradient:

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^{m} \left[\log D\left(\boldsymbol{x}^{(i)}\right) + \log\left(1 - D\left(G\left(\boldsymbol{z}^{(i)}\right)\right)\right)\right].$$

  end for
  • Sample minibatch of m noise samples $\{\boldsymbol{z}^{(1)}, \ldots, \boldsymbol{z}^{(m)}\}$ from noise prior $p_g(\boldsymbol{z})$.
  • Update the generator by descending its stochastic gradient:

$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^{m} \log\left(1 - D\left(G\left(\boldsymbol{z}^{(i)}\right)\right)\right).$$

end for
The gradient-based updates can use any standard gradient-based learning rule. We used momentum in our experiments.

Idea of convergence

1. Adversarial components → nontrivial convergence for the training

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1. Adversarial components $\rightarrow$ nontrivial convergence for the training
2. In other words, objective is not to push the loss/objective towards 0

Optimality

$$\min_G \max_D \left(\mathbb{E}_{x \sim p_{\text{data}}}[log D(x)] + \mathbb{E}_{z \sim p_z}[log(1 - D(G(z)))] \right)$$

$$\rightarrow \min_G \max_D \int_x \left(p_{\text{data}}(x) \cdot log D(x) + p_G(x) \cdot log(1 - D(x)) \right) dx$$

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let $y = D(x)$, $a = p_{\text{data}}$, and $b = p_G$

$\rightarrow f(y) = a \cdot \log y + b \cdot \log(1 - y)$

f exhibits local maximum at $y = \frac{a}{a+b}$

Optimal discriminator $D_G^*(x) = \frac{p_{\text{data}}(x)}{p_{\text{data}}(x) + P_G(x)}$

Optimality

$$\min_G \int_X \left(p_{\text{data}}(x) \cdot log D_G^*(x) + p_G(x) \cdot log(1 - D_G^*(x)) \right) dx$$

$$\min_G \int_X \left(p_{\text{data}}(x) \cdot \left[\log \frac{p_{\text{data}}(x)}{p_{\text{data}}(x) + P_G(x)} \right] + p_G(x) \cdot log(1 - \frac{p_{\text{data}}(x)}{p_{\text{data}}(x) + P_G(x)}) \right) dx$$

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$$\min_G \left(\mathbb{E}_{x \sim p_{\text{data}}} \left[\log \frac{p_{\text{data}}(x)}{p_{\text{data}}(x) + P_G(x)} \right] + \mathbb{E}_{x \sim p_G} \cdot log(\frac{p_{\mathsf{G}}(x)}{p_{\text{data}}(x) + P_G(x)}) \right)$$

Optimality

$$\min_G \left(\mathbb{E}_{x\sim p_{\text{data}}} \left[\log \frac{2 * p_{\text{data}}(x)}{2 * (p_{\text{data}}(x) + P_G(x))} \right] + \mathbb{E}_{x\sim p_G} \cdot log(\frac{2 * p_G(x)}{2 * (p_{\text{data}}(x) + P_G(x)})) \right)$$

$$\min_G \left(\mathbb{E}_{x\sim p_{\text{data}}} \left[\log \frac{2 * p_{\text{data}}(x)}{(p_{\text{data}}(x) + P_G(x))} \right] + \mathbb{E}_{x\sim p_G} \cdot log(\frac{2 * p_G(x)}{(p_{\text{data}}(x) + P_G(x)}) - \log 4) \right)$$

$$\min_G \left(\mathbf{KL}(\mathbf{p}_{\text{data}}(\mathbf{x}), \frac{\mathbf{p}_{\text{data}}(\mathbf{x}) + \mathbf{P_G}(\mathbf{x})}{\mathbf{2}}) + \mathbf{KL}(\mathbf{p}_{\mathsf{G}}(\mathbf{x}), \frac{(\mathbf{p}_{\text{data}}(\mathbf{x}) + \mathbf{P_G}(\mathbf{x})}{\mathbf{2}}) - \log \mathbf{4}) \right)$$

$$\min_G \left(2 * \mathbf{JSD}(\mathbf{p}_{\text{data}}, \mathbf{p_G}) - \log \mathbf{4} \right)$$

$\rightarrow$ minimized when $p_{\text{data}} = p_G$

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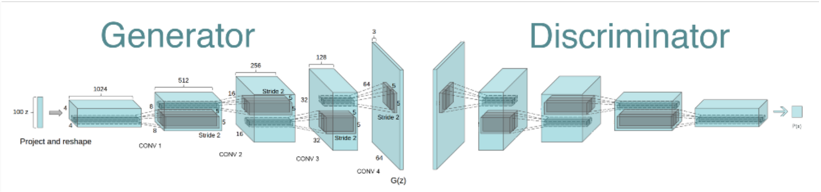
2. $p_{\text{data}} = p_G$ (Optimal Generator for any D)
3. $D_G^*(x) = \frac{1}{2}$

Deep Convolutional GAN (DC-GAN)



Radford et al. ICLR 2016

1. Combined the developments of CNNs with the generative modeling

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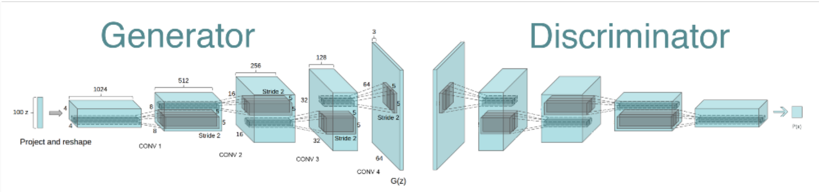

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1. Combined the developments of CNNs with the generative modeling
2. Demonstrated some of the best practices for stable training of deep GAN architectures

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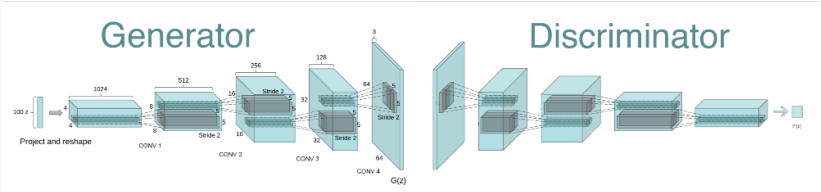

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1. Strided convolution in place of spatial pooling (learn spatial downsampling)

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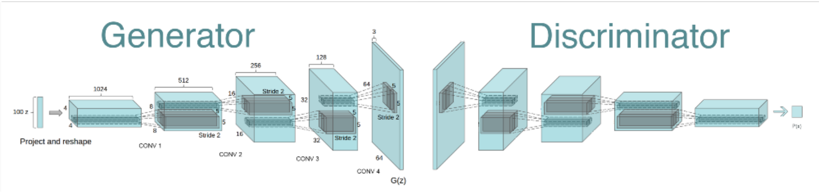

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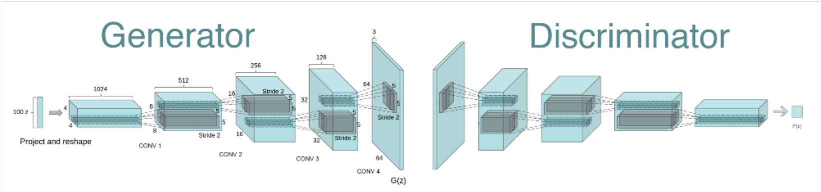

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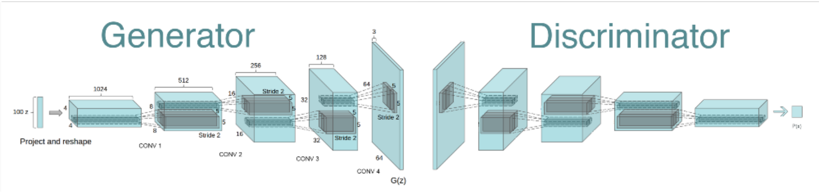

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1. Strided convolution in place of spatial pooling (learn spatial downsampling)
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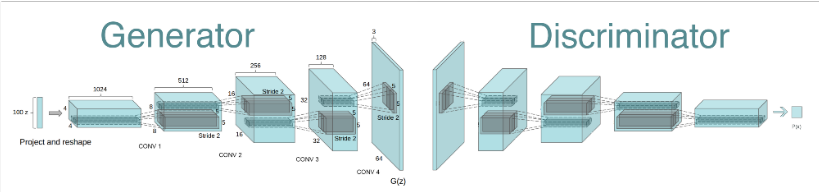
4. ReLU (tanh for the o/p layer) for G and Leaky-ReLU (sigmoid for the o/p layer) for D

Deep Convolutional GAN (DC-GAN)



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1. Smooth interpolation in the latent space and Vector arithmetic

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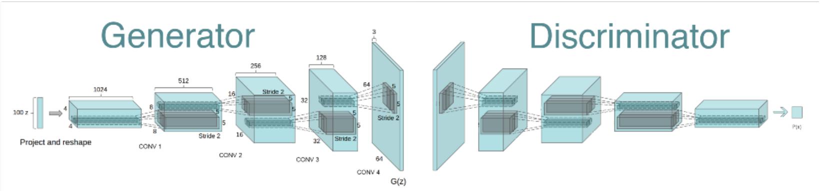

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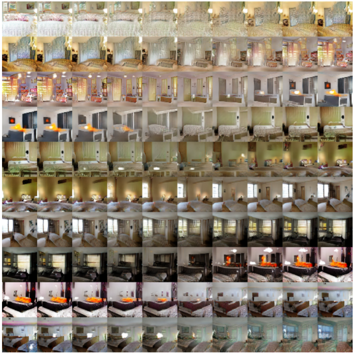
1. Smooth interpolation in the latent space and Vector arithmetic
2. Unsupervised feature learning (via the Discriminator)

Moving in the latent space

1. Interpolate between two points in the latent space and visualize



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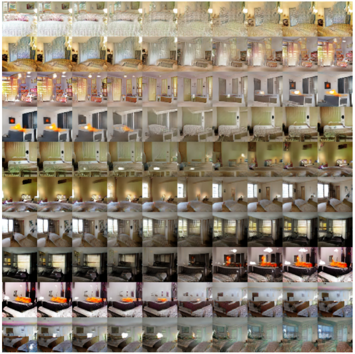
Moving in the latent space

1. Interpolate between two points in the latent space and visualize
2. Smooth transition in the generated image is a sign of good model



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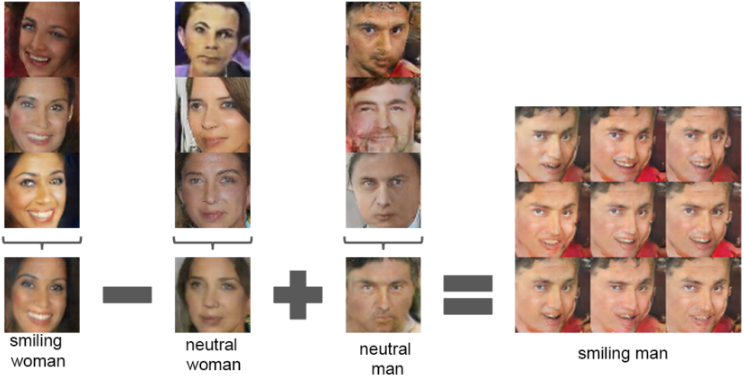
Vector arithmetic



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Pose Transformation

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Representation learning

Table 1: CIFAR-10 classification results using our pre-trained model. Our DCGAN is not pre-trained on CIFAR-10, but on Imagenet-1k, and the features are used to classify CIFAR-10 images.

Model	Accuracy	Accuracy (400 per class)	max # of features units
1 Layer K-means	80.6%	63.7% (±0.7%)	4800
3 Layer K-means Learned RF	82.0%	70.7% (±0.7%)	3200
View Invariant K-means	81.9%	72.6% (±0.7%)	6400
Exemplar CNN	84.3%	77.4% (±0.2%)	1024
DCGAN (ours) + L2-SVM	82.8%	73.8% (±0.4%)	512

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Evaluating GANs

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भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

1. Open research problem

Evaluating GANs

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2. Humans judgement!

Evaluating GANs

1. Open research problem
2. Humans judgement!
3. In case of images
   - **Recognizable objects**: accurate and high-confidence predictions by a classifier
   - **Semantic diversity**: samples should be drawn evenly from all categories of train data

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5. Higher is better

Inception Score (IS)

1. Based completely on the generated data (real data is not considered)

Frechet Inception Distance (FID)

1. Attempts to find the distance b/w p_{data} and p_G

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$$d^2((m, C), (m_d, C_d)) = |m - m_d|^2 + Tr(C + C_d - 2(C \cdot C_d)^2)$$

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