Deep Learning

17 Autoencoders

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Beyond Classification and Regression

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2. These applications require to learn the meaningful degrees of freedom that constitute the signal
3. Typically, these degrees of freedom are of lesser dimensions than the signal

Example: Synthesizing Human faces

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   - skull size and shape
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   - skull size and shape
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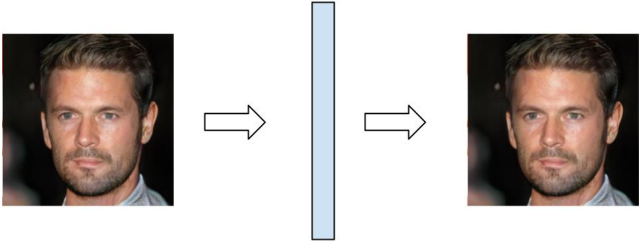
   - features of nose and lips, etc.
2. Even a comprehensive list of such things will be less than the number of pixels in the image (i.e. resolution)

Example: Synthesizing Human faces

1. If we can model these relatively small number of dimensions, we can synthesize a face with thousands of dimensions



Autoencoder: architecture

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3. Network consists of two parts: encoder (`f`) and decoder (`g`)

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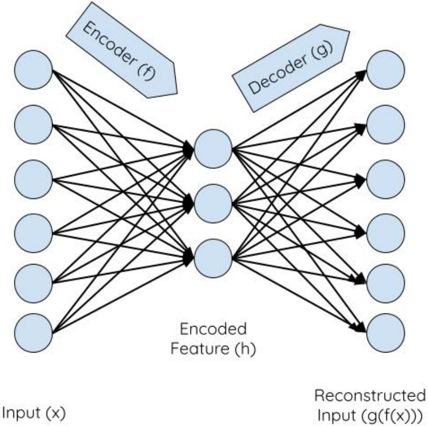

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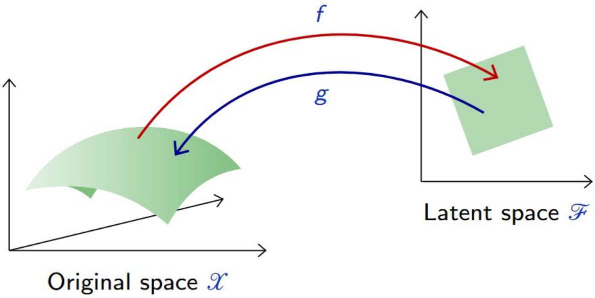

Figure credits: Francois Flueret

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3. Nonlinearity for `f`?

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2. Loss function takes care of this via training

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2. Training: finding the parameters for the encoder $(f(\cdot; w_f))$ and decoder $(g(\cdot; w_g))$ optimizing the empirical loss

$$\hat{w}_f, \hat{w}_g = \underset{w_f, w_g}{\operatorname{argmin}} \frac{1}{N} \sum_n \left\| x_n - g(f(x_n; w_f); w_g) \right\|^2$$

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2. Hence, we may use BCE loss for training

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2. Better results can be made possible with sophisticated transformations such as deep neural networks $\rightarrow$ Deep Autoencoders

Deep Autoencoders

Top row: original data samples
Bottom row: corresponding reconstructed samples (single ReLU layer of dimension 32)
Figure credits:Keras blog

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5. Tie the weights, i.e., $w_g = w_f^T$

Besides dimensionality reduction

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2. This can help to restore the missing components from an input

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Besides dimensionality reduction

భారతీయ సాంకేతిక విజ్ఞాన సంస్థ హైదరాబాద్
भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

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4. This is referred to as a **Denoising Autoencoder**

Denoising Autoencoder

1. This can be illustrated with an additive Gaussian noise in case of a 2D signal and MSE

$$\hat{w} = \underset{w}{\operatorname{argmin}} \frac{1}{N} \sum_{n=1}^{N} \|x_n - \phi(x_n + \epsilon_n; w)\|^2 ,$$

where x_n are data samples and ϵ_n are Gaussian random noise

Denoising Autoencoder

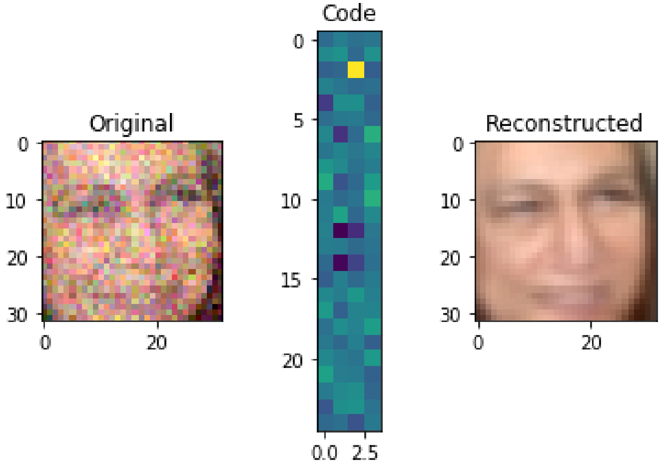

Figure credits: Ali Abdelal, https://stackabuse.com/

Sparse Autoencoders

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2. Restricts the freedom of the parameters by forcing them to fire sparsely
3. Uses a sparsity parameter (ρ) (typically close to 0, say 0.01)
4. Enforces the mean neuron activation $(\hat{\rho}_l)$ to be close to ρ

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3. k - dimension of hidden layer
   m -size of training dataset

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3. Competition (in the latent/hidden layer) b/w 'being sensitive' and 'not sensitive' to the i/p variations
4. Ends up capturing only the important variations in the i/p (something like PCA)

Latent Representations

1. Consider two samples in the latent space and reconstruct the samples along the line joining these

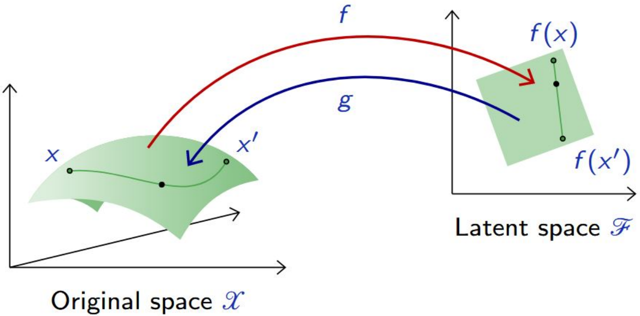

Figure credits: Francois Fleuret

Latent Representations

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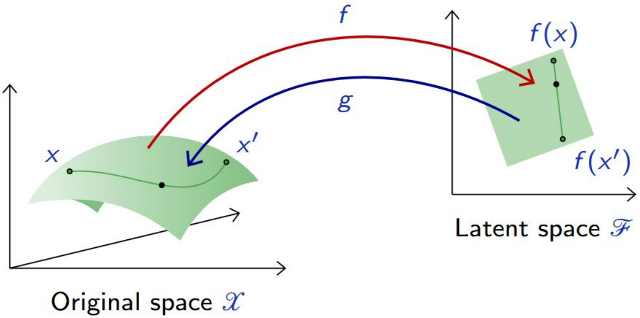
2. $g(\alpha x + (1-\alpha)x')$



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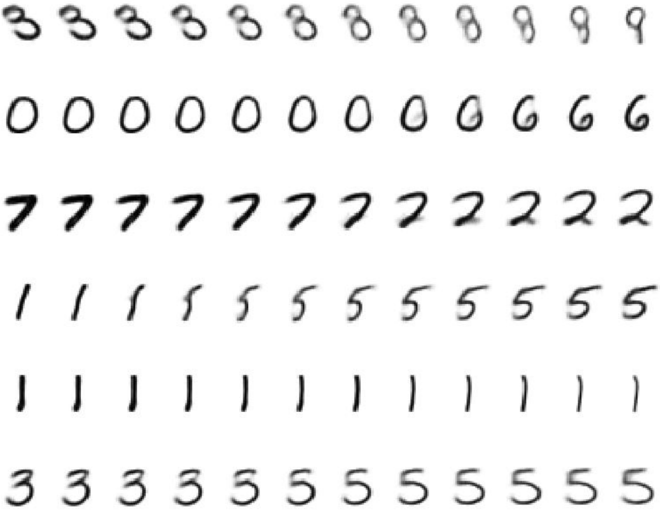

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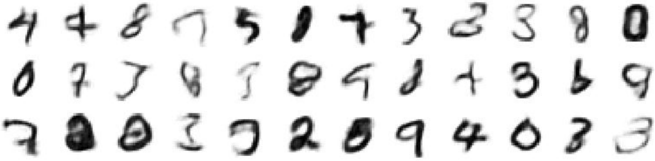
3. For instance, use a Gaussian density for modeling the latent space from the training data (estimate mean and a diagonal covariance matrix)

Generative Modeling by Autoencoder

Autoencoder sampling ($d = 8$)



Autoencoder sampling ($d = 16$)

Figure credits: Francois Fleuret

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3. Good model still needs to capture the empirical distribution on the data although in a lower dimensional space