Deep Learning

1 Artificial Neuron (MP Neuron and Perceptron)

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The Neuron

- About 100 billion neurons in human brain

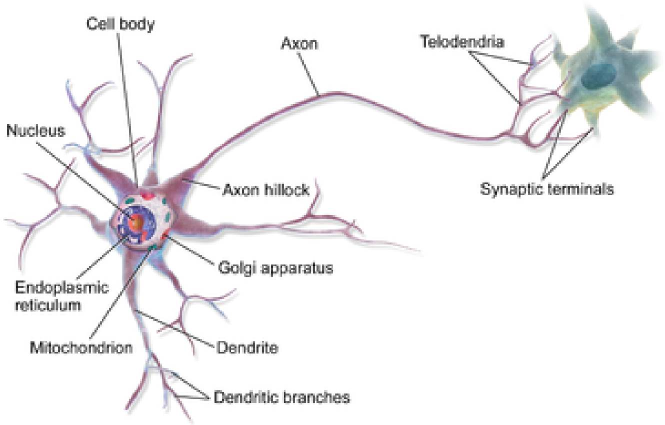

Figure credits: Wikipedia

Neuron in action

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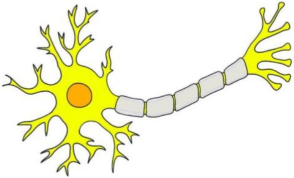

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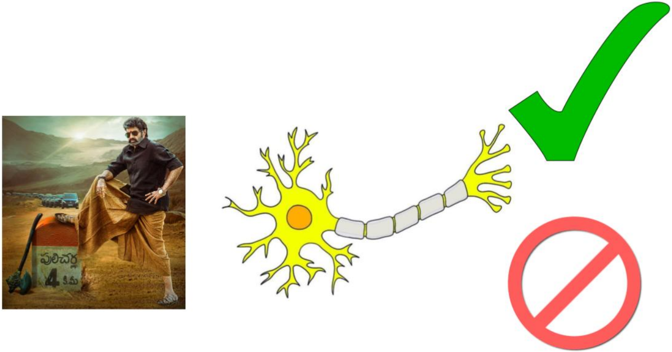

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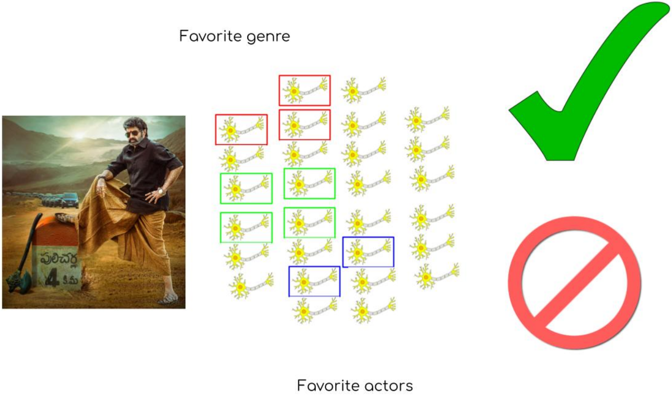
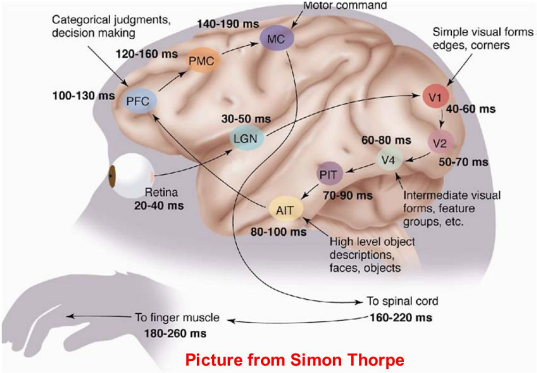


Neurons in the brain have a hierarchy



Threshold Logic Unit

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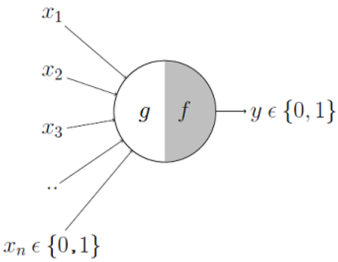

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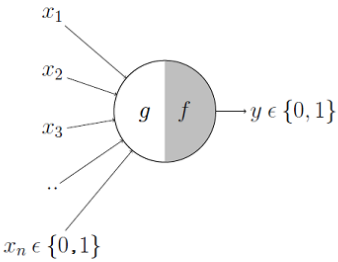

4.

$$f(x) = \mathbb{1}(\sum_i x_i \geq \theta)$$

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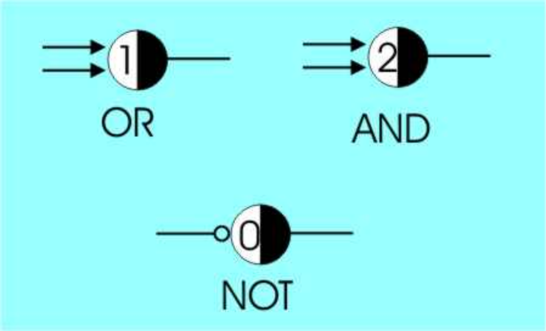
2. When an inhibitory input is set ($=1$) output $\rightarrow 0$
3. Counts the number of ‘ON’ signals on the excitatory inputs versus the inhibitory

Threshold Logic Unit



Example Boolean functions

Threshold Logic Unit

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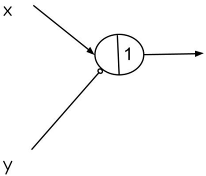

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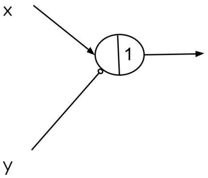
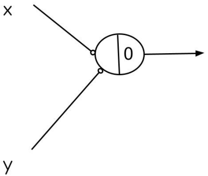


3. NOR



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2. No learning; heuristic approach

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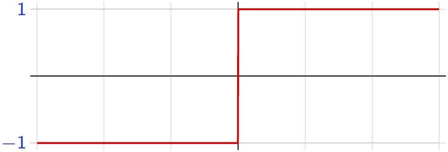
5. $$f(x) = \begin{cases} 1 & \text{when } \sum_i w_i x_i + b \geq 0 \\ 0 & \text{else} \end{cases}$$

Perceptron

1. For simplicity we consider +1 and -1 responses

$$\sigma(x) = \begin{cases} 1 & \text{when } x \geq 0 \\ -1 & \text{else} \end{cases}$$



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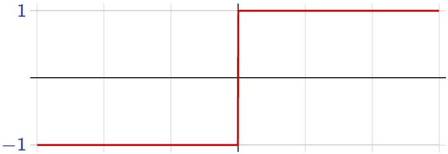

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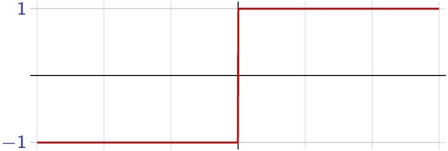

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3. $\mathbf{w}$ are referred to as weights and b as the bias

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4. Mechanism for learning weights

Weights and Bias

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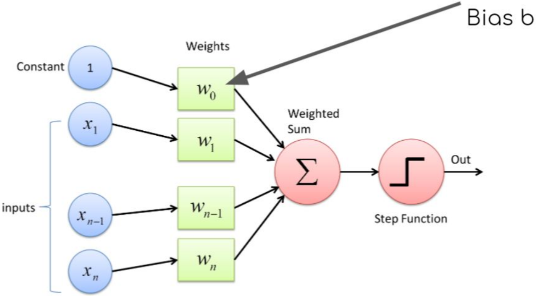
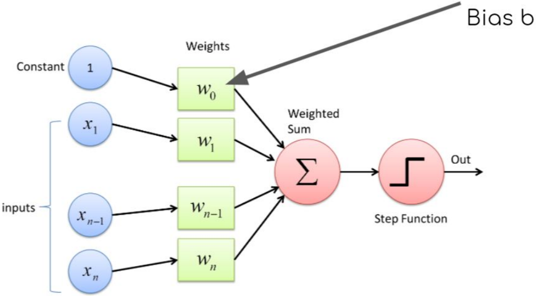


Figure credits: DeepAI

Weights and Bias

1. Why are the weights important?
2. Why is it called 'bias'? What does it capture?



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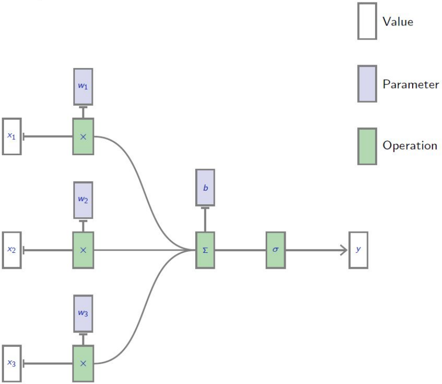

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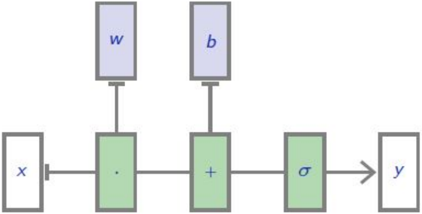
Perceptron



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Perceptron Learning algorithm

1. Training data $(x^i, y^i) \in \mathcal{R}^D \times \{-1, 1\}, i = 1, \ldots, N$

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भारतीय प्रौद्योगिकी संस्थान हैदराबाद
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   $\mathbf{w_{k+1}} = \mathbf{w_k} + \mathbf{y^i} \cdot \mathbf{x^i}$
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4. Note that the bias b is absorbed as a component of $\mathbf{w}$ and $\mathbf{x}$ is appended with 1 suitably

Perceptron Learning Algorithm

- Colab Notebook: Perceptron-learning

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3. Other algorithms maximize the margin from boundary to the samples